Optimal Placement of Sensor Networks for Target Localization and Tracking

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Outline

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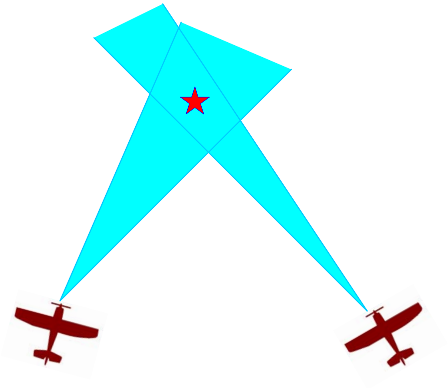
Problem Description

Cooperative target localization:



- Objective: localize the target
- Measurement: partial information of the target such as bearing or range
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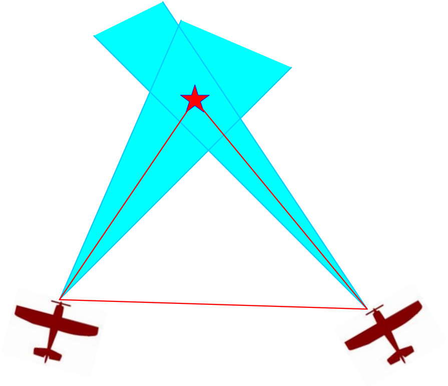

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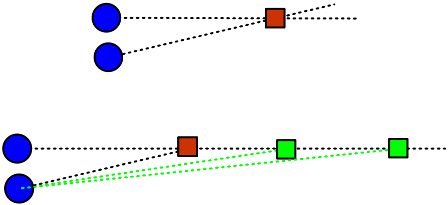

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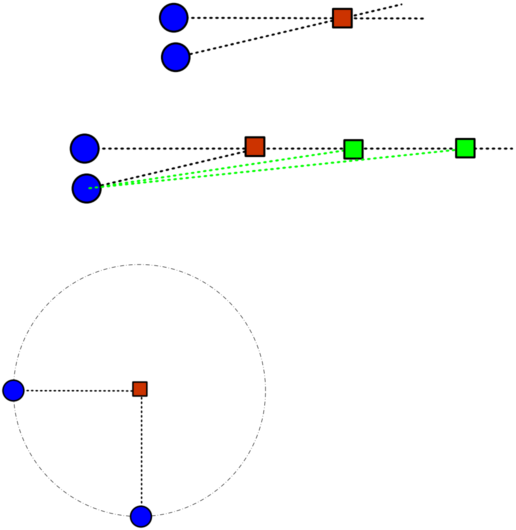

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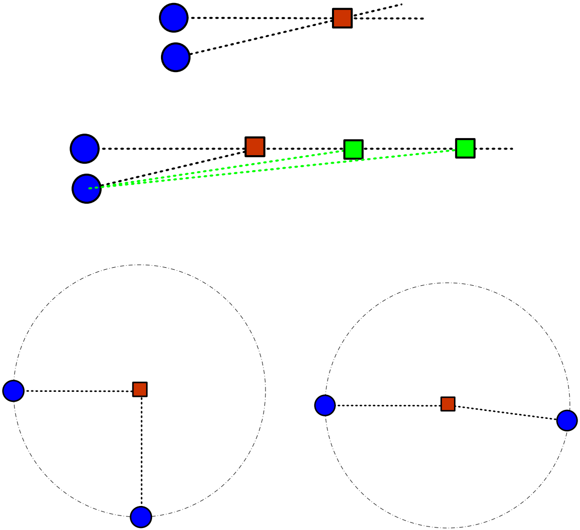

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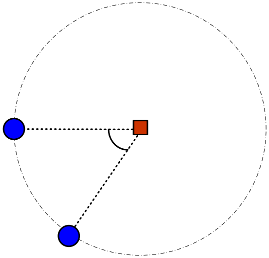
Numerical simulation: localization error and angle



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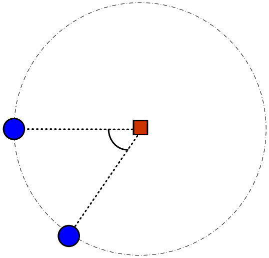

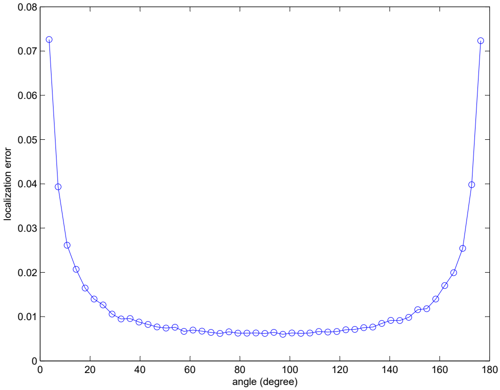

Literature Review

- **Sensor Types**
  - Bearing-only: Doğançay and Hmam [2008], Bishop et al. [2010]
  - Range-only: Martínez and Bullo [2006], Jourdan and Roy [2006], Bishop et al. [2010], Morales and Kassas [2015]
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- **Space Dimension**
  - 2D Space: Zhang [1995], Martínez and Bullo [2006], Jourdan and Roy [2006], Bishop et al. [2007], Doğançay [2007], Doğançay and Hmam [2008], Bishop and Jensfelt [2009], Isaacs et al. [2009], Bishop et al. [2010], Morales and Kassas [2015]
  - 3D Space: Moreno-Salinas et al. [2011]

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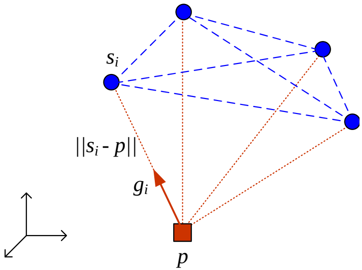
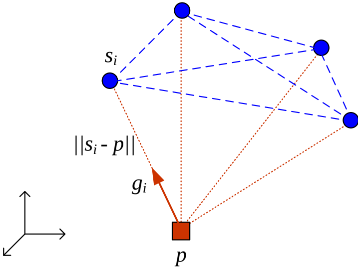
In our work

- Sensor type: bearing-only, range-only, and RSS-based sensors
- Space dimension: both 2-D and 3-D

Outline

Sensor Measurement Model





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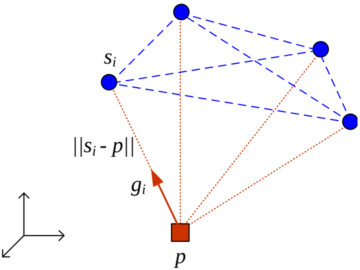

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- Range-only sensor: $h(s_i - p) = \|s_i - p\|$
- RSS-based sensor: $h(s_i - p) = \ln \|s_i - p\|$

Optimality Metrics

Fisher Information Matrix (FIM):

$$F = \sum_{i=1}^{n} \left[\frac{\partial h}{\partial p}\right]^T \Sigma_i^{-1} \frac{\partial h}{\partial p}$$

- Σ_i: how accurate the measurement is
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Common optimality metrics:

- T-Optimality: maximize $\operatorname{tr} F = \sum_{i=1}^{d} \lambda_i$
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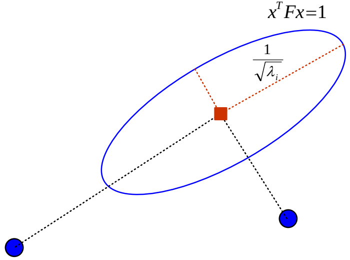

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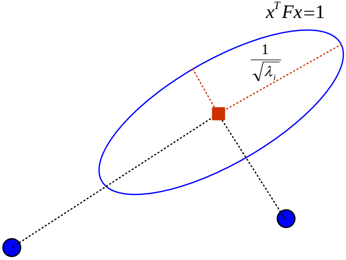

Limitation: not applicable to 3-D cases

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Connection between the new and old metrics:

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- 3-D case:
  - case 1: $\det F = \bar{\lambda}^3$ iff $\|F - \bar{\lambda} I_3\| = 0$ (equivalent)

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The new optimality metric:

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- 3-D case:
  - case 1: $\det F = \bar{\lambda}^3$ iff $\|F - \bar{\lambda} I_3\| = 0$ (equivalent)
  - case 2: $\det F < \bar{\lambda}^3$ and $\|F - \bar{\lambda} I_d\| > 0$ (not equivalent)

Problem Statement

For bearing-only, range-only, and RSS sensors

$$\min \|F - \bar{\lambda} I_d\|^2 \Longleftrightarrow \min \|G\|^2$$

where

$$G = \sum_{i=1}^{n} c_i^2 g_i g_i^T$$

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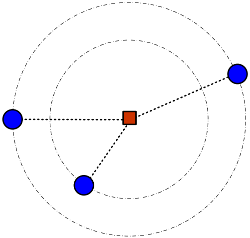

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Necessary and Sufficient Condition for Optimal Placement

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c_i is large when σ_i is small (the measurement is accurate) or $\|r_i\|$ is small

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Definition (Irregularity)

- The weights for n sensors: $c_1 \geq c_2 \geq \cdots \geq c_n > 0$
- Dimension: $d = 2, 3$

Denote k_0 as the smallest nonnegative integer k for which

$$c_{k+1}^2 \leq \frac{1}{d-k} \sum_{i=k+1}^{n} c_i^2.$$

The integer k_0 is called the irregularity of $\{c_i\}_{i=1}^{n}$ with respect to d.

Regular VS Irregular

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Example (Regular)

$\{c_i\}_{i=1}^{4} = \{1, 1, 1, 1\}$ & $d = 3$:

- $k = 0$: $1 = c_1^2 \leq \frac{1}{d} \sum_{i=1}^{n} c_i^2 = 1.3$

Irregularity=0

Regular VS Irregular

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Example (Irregular)

$\{c_i\}_{i=1}^4 = \{10, 1, 1, 1\}$ & $d = 3$:

- $k = 0$: $100 = c_1^2 > \frac{1}{d} \sum_{i=1}^n c_i^2 = 34.3$
- $k = 1$: $1 = c_2^2 \leq \frac{1}{d-1} \sum_{i=2}^n c_i^2 = 1.5$

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- $k = 1$: $100 = c_2^2 > \frac{1}{d-1}\sum_{i=2}^n c_i^2 = 51$
- $k = 2$: $1 = c_2^2 \leq \frac{1}{d-2}\sum_{i=3}^n c_i^2 = 2$

Irregularity=2

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$\{c_i\}_{i=1}^4 = \{10,10,1,1\}$ & $d = 3$:

- $k = 0$: $100 = c_1^2 > \frac{1}{d}\sum_{i=1}^n c_i^2 = 67.3$
- $k = 1$: $100 = c_2^2 > \frac{1}{d-1}\sum_{i=2}^n c_i^2 = 51$
- $k = 2$: $1 = c_2^2 \leq \frac{1}{d-2}\sum_{i=3}^n c_i^2 = 2$

Irregularity=2

Intuition

A sequence is irregular when certain element is much larger than the others

Necessary and Sufficient Condition for Optimal Placement

Theorem (Regular optimal placement)

In $\mathbb{R}^d$ with $d = 2$ or 3, if the positive coefficient sequence $\{c_i\}_{i=1}^n$ is regular, then the objective function $\|G\|^2$ satisfies

$$\|G\|^2 \geq \frac{1}{d}\left(\sum_{i=1}^{n} c_i^2\right)^2.$$

The lower bound of $\|G\|^2$ is achieved if and only if

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Theorem (Irregular optimal placement)

In $\mathbb{R}^d$ with $d = 2$ or 3, if the positive coefficient sequence $\{c_i\}_{i=1}^n$ is irregular with irregularity as $k_0 \geq 1$, without loss of generality $\{c_i\}_{i=1}^n$ can be assumed to be a non-increasing sequence, and then the objective function $\|G\|^2$ satisfies

$$\|G\|^2 \geq \sum_{i=1}^{k_0} c_i^4 + \frac{1}{d-k_0}\left(\sum_{i=k_0+1}^n c_i^2\right)^2.$$

The lower bound of $\|G\|^2$ is achieved if and only if

$$\{g_i\}_{i=1}^n = \{g_i\}_{i=1}^{k_0} \cup \{g_i\}_{i=k_0+1}^n,$$

where $\{g_i\}_{i=1}^{k_0}$ is an orthogonal set, and $\{g_i\}_{i=k_0+1}^n$ forms a regular optimal placement in the $(d-k_0)$-dimensional orthogonal complement of $\{g_i\}_{i=1}^{k_0}$.

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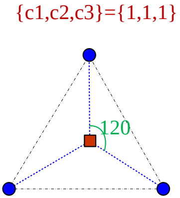
Observation

An **irregular** optimal sensor placement problem can be converted to a **regular** optimal sensor placement in a **lower dimensional space**

Necessary and Sufficient Condition for Optimal Placement

2-D example:



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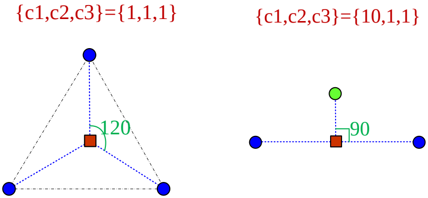

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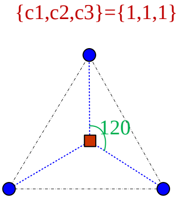
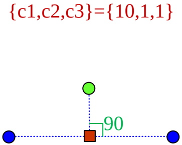
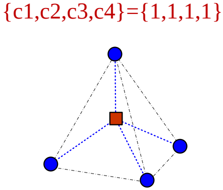




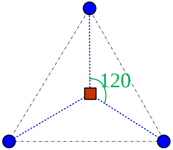
3-D example:



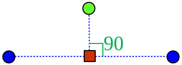
Necessary and Sufficient Condition for Optimal Placement

2-D example:

{c1,c2,c3}={1,1,1}



{c1,c2,c3}={10,1,1}



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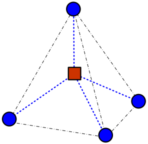

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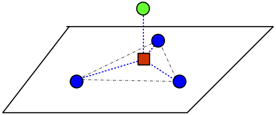

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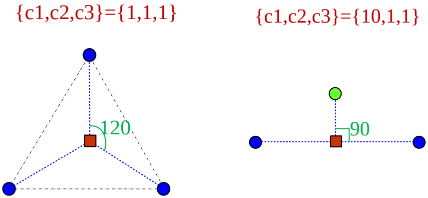

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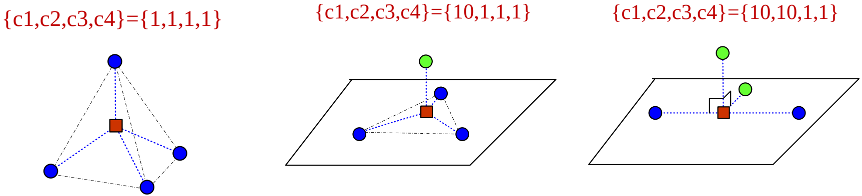

Outline

Other Interesting Properties

Equally-weighted sensor networks:

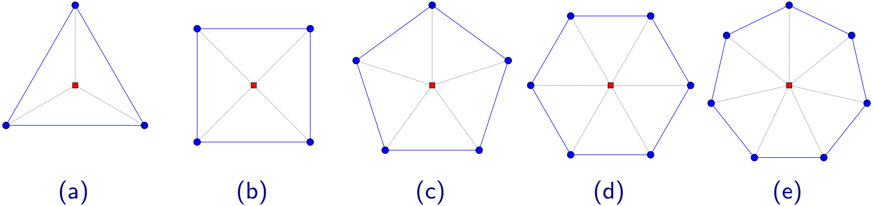

Figure: Examples of 2D equally-weighted optimal placements

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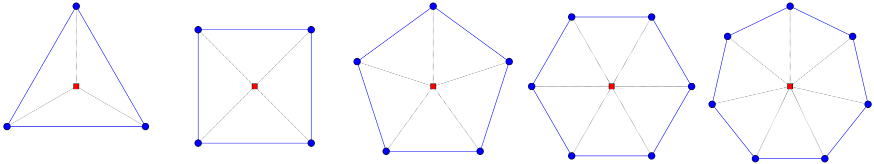

(a) (b) (c) (d) (e)

Figure: Examples of 2D equally-weighted optimal placements

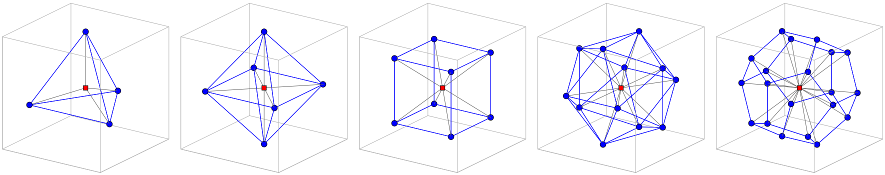

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Figure: Examples of 3D equally-weighted optimal placements

Other Interesting Properties

Theorem (Distributed construction)

The union of multiple disjoint regular optimal placements in $\mathbb{R}^d$ *(*$d = 2$ *or* 3*) is still a regular optimal placement in* $\mathbb{R}^d$*.*

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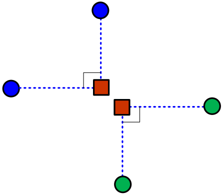

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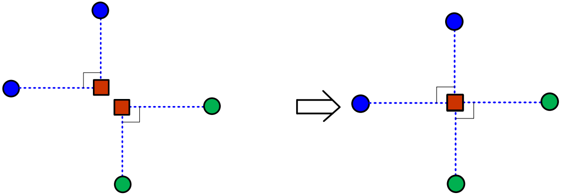

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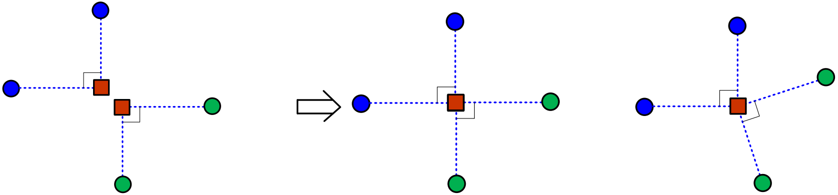

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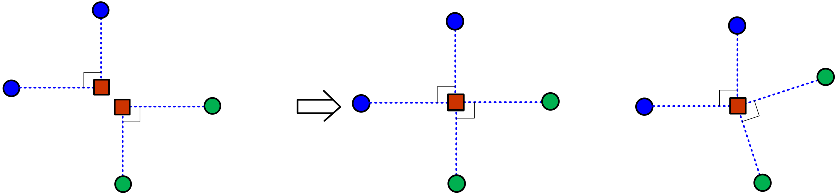

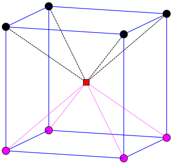

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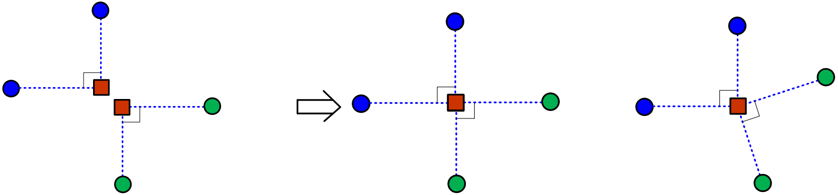

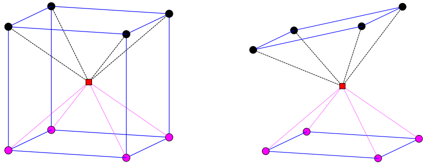

Autonomous Optimal Sensor Deployment

$V = \|G\|^2/4$. Gradient descent control:

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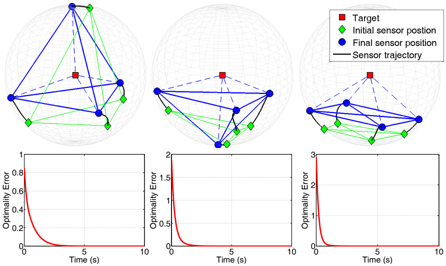

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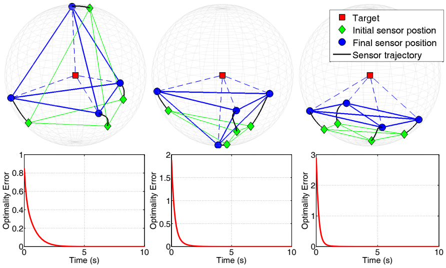
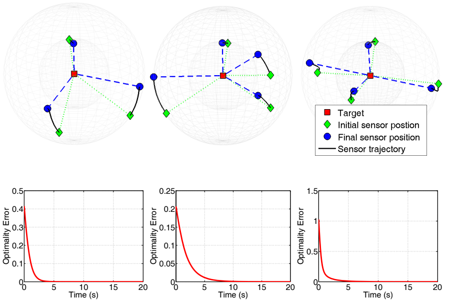
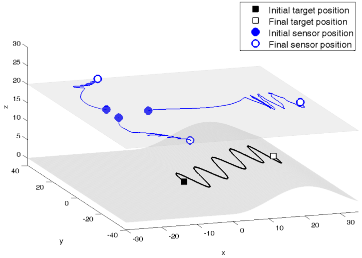


Irregular placements:



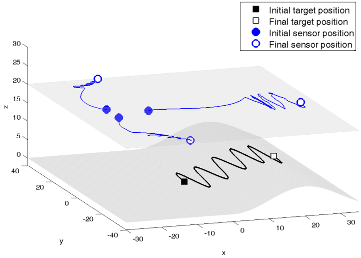
(g) $n = 3$, $k_0 = 1$ (h) $n = 4$, $k_0 = 1$ (i) $n = 4$, $k_0 = 2$

Autonomous Optimal Sensor Deployment



(a) Trajectory

Autonomous Optimal Sensor Deployment



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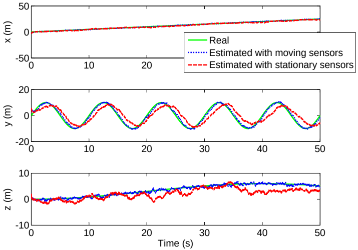

(b) Estimation error

Outline

Conclusions

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Future work:

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Future work:

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The End

Q & A

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