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Event-triggered cooperative unscented Kalman filtering and its application in multi-UAV systems[☆]

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ABSTRACT

This paper proposes a novel consensus-based distributed unscented Kalman filtering algorithm with event-triggered communication mechanisms. With such an algorithm, each sensor node transmits the newest measurement to the corresponding remote estimator selectively on the basis of its own event-triggering condition. Compared to the existing approaches, the proposed algorithm can significantly reduce unnecessary data transmissions and hence save communication energy consumption and alleviate the communication burden. A sufficient condition is provided to guarantee the stochastic stability of the distributed nonlinear filtering scheme. The proposed algorithm is applicable to a wide range of distributed estimation tasks, e.g., tracking a moving target with multiple unmanned aerial vehicles (UAVs). Simulation results demonstrate the feasibility and effectiveness of the proposed filtering algorithm.

1. Introduction

The past decades have witnessed ever-increasing research attention devoted to the sensor networks due to its extensive applications in many fields such as battlefield detection, environment monitoring, information processing, autonomous navigation, target tracking and localization. One of the most important problems in sensor networks is designing the functional filtering algorithms to estimate the state of the target process (Olfati-Saber, 2009). Compared with the centralized setting, distributed state estimation without requiring information center has several advantages including stronger fault-tolerance, less computational and communication loads. The central issue in distributed state estimation is to cooperatively estimate the states of a dynamic system via a wireless sensor network with given communication topology. Specifically, each node in such a distributed framework only needs to share information with its neighbors over networks. As a popular approach to address distributed state estimation problem, consensus-based methodologies have made significant progress in recent years (Battistelli & Chisci, 2014, 2016; Battistelli, Chisci, Mugnai, & Farina, 2014; Ji, Lewis, Hou, & Mikulski, 2017; Kamal, Farrell, & Roy-Chowdhury, 2013; Li, Jia, & Du, 2016; Matei & Baras, 2012; Olfati-Saber, 2007, 2009; Shen, Wang, & Hung, 2010). In Olfati-Saber (2007, 2009), Kalman consensus filters (KCF) are proposed to achieve a consensus in terms of the local state estimates by way of adding a consensus term, which is also applicable to the case with packet dropout. In Matei and Baras (2012), a Luenberger-like consensus algorithm is developed, where every sensor combines its own local estimate computed by the Luenberger observer and the other estimates obtained from its neighboring nodes in a convex manner. In addition, for the uncertain systems, H_∞-consensus performance constraint is introduced in Shen et al. (2010) to quantify the consensus level with regard to the estimation errors.

However, when there exist some non-ideal conditions, such as noisy transmission channel, restricted communication network or limited observability (Ji et al., 2017), the state estimation may be a more troublesome and challenging problem. To overcome these difficulties, an information-weighted consensus filter (ICF) algorithm is discussed in Ji et al. (2017), Kamal et al. (2013). In Battistelli et al. (2014), the stability of the hybrid CMCI (consensus on measurement (Olfati-Saber, 2007) and consensus on information (Battistelli & Chisci, 2014)) filtering algorithm based on the collective observability and network connectivity condition is guaranteed in a linear setting, which is equivalent to ICF with particular weights. When it comes to the nonlinear

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systems, an alternative extended Kalman filter (EKF) argument is raised in Battistelli and Chisci (2016), Battistelli et al. (2014), Hu, Wang, Gao, and Stergioulas (2012), Li et al. (2016). Similar to Battistelli et al. (2014), the local stability analysis of distributed extended Kalman filter (DEKF) under certain conditions is provided in Battistelli and Chisci (2016). In Li et al. (2016), a variance-constrained DEKF is put forward without omitting the edge-covariances in Olfati-Saber (2009), and the filter gain is obtained by minimizing an upper bound for the estimation error covariance. However, the EKF-based algorithm suffers a number of limitations especially when the system contains high nonlinearities and even discontinuities, which facilitates the development of unscented Kalman filter (UKF)-based algorithm (Julier & Uhlmann, 2004; Li, Wei, & Han, 2015; Li, Wei, Han, & Liu, 2016; Li & Xia, 2012; Xiong, Zhang, & Chan, 2006). In Li et al. (2015), a distributed UKF algorithm based on CI method is proposed, and lately, the weighted average consensus-based UKF is developed with theoretical proof in Li et al. (2016).

In many practical applications of sensor networks, the sensors are battery-powered, which brings about an inevitable issue that replacing or recharging the worn batteries might be impossible in a complicated environment. Thus, it is of particular importance to decrease the frequency of sensor-to-estimator data transmission without compromising the expected estimation performance (Miskowicz, 2006; Wu, Jia, Johansson, & Shi, 2013). A large number of related works considering event-triggered communication mechanism have been reported in Dimarogonas, Frazzoli, and Johansson (2012), Li, Jia, and Du (2016), Li, Yu, Xia, and Yang (2017), Liu, Wang, He, and Zhou (2015), Shi, Chen, and Shi (2014), Trimpe (2014), Zhang and Jia (2017), Zhang, Kuai, Ren, Luo, and Lin (2016), Zheng and Fang (2016). Based on the KCF framework in Olfati-Saber (2009), a kind of event-triggered KCF is derived with a named send-on-delta (SoD) schedule (Miskowicz, 2006) on estimator-to-estimator channel to reduce communication energy consumption in Li et al. (2016). An extended work can be found in Zhang and Jia (2017), where the sensor-to-estimator channel is also taken into consideration. However, to the best of our knowledge, there have been very few results about event-triggered UKF algorithm except Li et al. (2017) with only one single sensor considered, even none in distributed setting.

In addition to the theoretical developments, the work presented here is applied in the moving target tracking problem for a team of UAVs equipped with onboard sensors. Mobile UAV sensing platforms have attracted increasing attention in recent years due to the distinctive advantages over their static counterparts with regard to the area coverage, estimation performance and robustness against failure (Campbell & Whitacre, 2007; Hausman, Mueller, & Hariharan, 2015; Morbidi & Mariottini, 2013). In Zhan, Casbeer, and Swindlehurst (2010), a centralized adaptive target-tracking algorithm based on the UAV sensors is developed in the EKF framework, which is further investigated in the case where a maneuvering target is tracked with distributed UKF (Li & Jia, 2012) and distributed high degree cubature information filter (Sun & Xin, 2015) in the multiple model environment, respectively. However, the power constraints on small UAVs will limit the practicality of the existing algorithms, in which the data transmissions between individuals and ground stations or among individuals are executed in a periodical fashion. Therefore, the proposed event-triggered cooperative algorithm will be utilized in this application to reach a balance between tracking performance and energy consumption.

The main contributions of this paper are threefold:

(1) an event-triggered cooperative UKF algorithm is derived, which can well balance the filtering performance and average communication rate by designing reasonable trigger thresholds.
(2) the stochastic stability of the proposed algorithm in terms of the bounded estimation errors is investigated based on the stochastic stability theory.
(3) the proposed algorithm is employed in the moving target localization problem with multiple UAVs to show the practical potentials.

The remainder of this paper is organized as follows. Section 2 provides some basic concepts in algebraic graph theory and the mathematical formulation of the considered problem. Section 3 derives the event-triggered cooperative unscented Kalman filtering algorithm, whose stochastic stability will be analyzed in Section 4. Section 5 presents the simulation results illustrating the performance of proposed algorithm for the ground moving target localization with multiple UAVs tracking system. Concluding remarks are stated in Section 6.

2. Preliminaries and problem formulation

First of all, we present some notations that will be used throughout this paper. Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ be a real n-dimensional Euclidean vector space and a real $n \times m$ matrix space, respectively. Let $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ be the largest and the smallest eigenvalues of a real matrix. $\mathbb{E}\{\cdot\}$ is the expectation operation. $\|\cdot\|$ represents the Euclidean norm in $\mathbb{R}^n$. For a matrix A, A^T and A^{-1} denote its transpose and inverse, respectively. $\mathrm{tr}\{A\}$ denotes the trace of A and $A > 0$ means A is a positive definite matrix. $\mathrm{diag}(A_1, A_2, \dots, A_n)$ refers to the diagonal matrix where $A_1, A_2, \dots, A_n$ are the main diagonal matrix blocks. $\mathrm{col}(\cdot)$ denote the operation to aggregate all the column vectors into a single column vector. 1_N denotes the all-1 vector with dimension N. I_n is the $n\times n$ identity matrix. The subscript n will be dropped if the dimension is clear from the context.

2.1. Communication graph

The communication topology of the sensor network can be represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with a finite vertex set $\mathcal{V} = \{1, 2, \dots, N\}$, corresponding to the N sensor nodes, and an edge set $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$, corresponding to the communication channels between sensor nodes. If there exists an edge $(j, i) \in \mathcal{E}$, then node i and node j are said to be adjacent and node i can receive information from node j. For an undirected graph, $(j, i) \in \mathcal{E}$ if and only if $(i, j) \in \mathcal{E}$. The set of neighbors of node i is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$, which contains the nodes that node i can communicate with.

2.2. Problem formulation

Consider a nonlinear discrete-time system described by

$$x_k = f(x_{k-1}) + \omega_{k-1} \tag{1}$$

$$z^i_k = h^i(x_k) + \nu^i_k, \qquad i = 1, 2, \dots, N, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector at discrete-time instant k, and $z^i_k \in \mathbb{R}^m$ is the measurement vector of the ith sensor. $f : \mathbb{R}^n \to \mathbb{R}^n$ and $h^i : \mathbb{R}^n \to \mathbb{R}^m$ denote the nonlinear process function and the ith measurement equation, respectively. The process noise $\omega_{k-1} \in \mathbb{R}^n$ and the ith measurement noise $\nu^i_k \in \mathbb{R}^m$ are assumed to be mutually uncorrelated zero-mean Gaussian white noise sequences with the respective covariance matrices $Q_{k-1} \in \mathbb{R}^{n\times n}$ and $R^i_k \in \mathbb{R}^{m\times m}$. Assume that the target plant is detected by N battery-powered sensors, which transmit data to the corresponding remote state estimators through wireless channels. In order to reduce communication costs and expand sensors' service life, each sensor i is equipped with an event-triggered scheduler that decides whether to allow a measurement data transmission. The widely used SoD strategy (Miskowicz, 2006) is employed herein. Specifically, at each time instant k, sensor i outputs a new local measurement update z_k^i, and the scheduler of node i determines whether to send the updated value to its estimator according to the following event-triggered condition:

$$\gamma_k^i = \begin{cases} 1, & \text{if } (z_k^i - z_{\tau_{k-1}^i}^i)^T (z_k^i - z_{\tau_{k-1}^i}^i) > \delta^i, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

where τ_{k-1}^i is the last measurement transmitted time instant of sensor i, and $\delta^i > 0$ is the predetermined trigger threshold. The update rule of τ_k^i is described as

$$\tau_k^i = \begin{cases} k, & \text{if } \gamma_k^i = 1, \\ \tau_{k-1}^i, & \text{if } \gamma_k^i = 0, \end{cases} \tag{4}$$

namely,

$$z_{\tau_k^i}^i = \gamma_k^i z_k^i + (1-\gamma_k^i) z_{\tau_{k-1}^i}^i. \tag{5}$$

3. Event-triggered cooperative unscented Kalman filtering algorithm

In this section, the event-triggered cooperative unscented Kalman filtering algorithm will be derived. Similar to Li et al. (2017), for each estimator node $i \in \mathcal{V}$, the state estimate update equation is defined by

$$\begin{aligned} \hat{x}_{k+1}^i &= \hat{x}_{k+1|k}^i + \gamma_{k+1}^i K_{k+1}^i (z_{k+1}^i - \hat{z}_{k+1|k}^i) \\ &\quad + (1-\gamma_{k+1}^i) M_{k+1}^i (z_{\tau_k^i}^i - \hat{z}_{k+1|k}^i). \end{aligned} \tag{6}$$

For the convenience of further analysis, the prediction error and estimation error for node i at time instant $k+1$ are respectively defined as follows

$$\tilde{x}_{k+1|k}^i = x_{k+1} - \hat{x}_{k+1|k}^i, \tag{7}$$

$$\tilde{x}_{k+1}^i = x_{k+1} - \hat{x}_{k+1}^i, \tag{8}$$

where $\hat{x}_{k+1|k}^i$ denotes the *a priori* estimate of node i at time $k+1$, and $\hat{x}_{k+1}^i$ denotes the *a posteriori* estimate of node i at time $k+1$. The matrices $K_{k+1}^i \in \mathbb{R}^{n\times m}$ and $M_{k+1}^i \in \mathbb{R}^{n\times m}$ are the estimator gains to be determined next.

In fact, the estimate update equation (6) downgrades to the standard update form of UKF (Julier & Uhlmann, 2004) if $\gamma_{k+1}^i = 1$. In this case, the estimator gain matrix K_{k+1}^i is defined as

$$K_{k+1}^i = P_{x_{k+1}z_{k+1}}^i (P_{z_{k+1}z_{k+1}}^i)^{-1} \tag{9}$$

and the estimation error covariance P_{k+1}^i is updated by

$$P_{k+1}^i = P_{k+1|k}^i - K_{k+1}^i P_{z_{k+1}z_{k+1}}^i (K_{k+1}^i)^T \tag{10}$$

where $P_{k+1|k}^i$, $P_{z_{k+1}z_{k+1}}^i$ and $P_{x_{k+1}z_{k+1}}^i$ denote the predicted covariance, predicted measurement covariance, and state-measurement cross-covariance. To derive the gain matrix M_{k+1}^i and the update rule of the estimation error covariance when $\gamma_{k+1}^i = 0$, a lemma is required beforehand.

Lemma 1 (Hu et al., 2012). *For any given $x, y \in \mathbb{R}^n$, and scalar $\sigma > 0$, one has*

$$xy^T + yx^T \le \sigma xx^T + \sigma^{-1} yy^T. \tag{11}$$

Considering the case where $\gamma_{k+1}^i = 0$, the estimate update equation (6) can be rewritten by the following:

$$\hat{x}_{k+1}^i = \hat{x}_{k+1|k}^i + M_{k+1}^i (z_{\tau_k^i}^i - z_{k+1}^i) + M_{k+1}^i (z_{k+1}^i - \hat{z}_{k+1|k}^i). \tag{12}$$

Substituting (12) into (8) and using (7) yields

$$\tilde{x}_{k+1}^i = \tilde{x}_{k+1|k}^i - M_{k+1}^i (z_{\tau_k^i}^i - z_{k+1}^i) - M_{k+1}^i (z_{k+1}^i - \hat{z}_{k+1|k}^i). \tag{13}$$

Assume that the functions $h^i(\cdot)$ are continuously differentiable at $\hat{x}_{k+1|k}^i$, using the Taylor expansion, we obtain

$$h^i(x_{k+1}) = h^i(\hat{x}_{k+1|k}^i) + H_{k+1}^i \tilde{x}_{k+1|k}^i + \phi^i(x_{k+1}, \hat{x}_{k+1|k}^i), \tag{14}$$

where $H_{k+1}^i = \frac{\partial h^i(x_{k+1})}{\partial x_{k+1}}\Big|_{x_{k+1}=\hat{x}_{k+1|k}^i}$. Following the derivation of EKF in Battistelli and Chisci (2016), Kluge, Reif, and Brokate (2010), the event-triggered single UKF in Li et al. (2017) and the unscented information filtering in Lee (2008), the linearization technique is utilized herein, i.e., only the first-order term in (14) is reserved in the subsequent analysis. Then noticing that $\hat{z}_{k+1|k}^i = h^i(\hat{x}_{k+1|k}^i)$ in (13) yields

$$\tilde{x}_{k+1}^i = A_{k+1}^i \tilde{x}_{k+1|k}^i - M_{k+1}^i \rho_{k+1}^i - M_{k+1}^i v_{k+1}^i, \tag{15}$$

where $A_{k+1}^i = I - M_{k+1}^i H_{k+1}^i$ and $\rho_{k+1}^i = z_{\tau_k^i}^i - z_{k+1}^i$.

Next, the estimation error covariance P_{k+1}^i is calculated by

$$\begin{aligned} &P_{k+1}^i \\ &= \mathbb{E}\{\tilde{x}_{k+1}^i (\tilde{x}_{k+1}^i)^T\} \\ &= A_{k+1}^i P_{k+1|k}^i (A_{k+1}^i)^T - \mathcal{M}_{k+1}^i - (\mathcal{M}_{k+1}^i)^T - \mathcal{N}_{k+1}^i \\ &\quad - (\mathcal{N}_{k+1}^i)^T + M_{k+1}^i \mathbb{E}\{\rho_{k+1}^i (\rho_{k+1}^i)^T\} (M_{k+1}^i)^T \\ &\quad + M_{k+1}^i \mathbb{E}\{v_{k+1}^i (v_{k+1}^i)^T\} (M_{k+1}^i)^T + \mathcal{O}_{k+1}^i + (\mathcal{O}_{k+1}^i)^T, \end{aligned} \tag{16}$$

where $\mathcal{M}_{k+1}^i = A_{k+1}^i \mathbb{E}\{\tilde{x}_{k+1|k}^i (\rho_{k+1}^i)^T\} (M_{k+1}^i)^T$, $\mathcal{N}_{k+1}^i = A_{k+1}^i \mathbb{E}\{\tilde{x}_{k+1|k}^i (v_{k+1}^i)^T\} (M_{k+1}^i)^T$ and $\mathcal{O}_{k+1}^i = M_{k+1}^i \times \mathbb{E}\{\rho_{k+1}^i (v_{k+1}^i)^T\} (M_{k+1}^i)^T$.
It can be verified that the term $\mathcal{N}_{k+1}^i$ is equal to zero since neither $\tilde{x}_{k+1|k}^i$, M_{k+1}^i nor H_{k+1}^i depend on v_{k+1}^i. Note that the additional terms, involving ρ_{k+1}^i, are induced by the event-triggered transmission mechanism, which will make it difficult to design the filter gain M_{k+1}^i.

By applying Lemma 1 to the terms $(-\mathcal{M}_{k+1}^i - (\mathcal{M}_{k+1}^i)^T)$ and $(-\mathcal{O}_{k+1}^i - (\mathcal{O}_{k+1}^i)^T)$ respectively and using (3) as in Li et al. (2017), the following inequality holds

$$\begin{aligned} P_{k+1}^i &\le \eta_1 A_{k+1}^i P_{k+1|k}^i (A_{k+1}^i)^T + \eta_2 M_{k+1}^i R_{k+1}^i (M_{k+1}^i)^T \\ &\quad + \eta_3 M_{k+1}^i \delta^i (M_{k+1}^i)^T. \end{aligned} \tag{17}$$

where $\eta_1 = 1+\sigma_1$, $\eta_2 = 1+\sigma_2$, $\eta_3 = 1+\sigma_1^{-1}+\sigma_2^{-1}$, and σ_1, σ_2 are positive scalars. Define the terms of the right-hand side of (17) as $\bar{P}_{k+1}^i$, namely, an upper bound for the filtering error covariance P_{k+1}^i. According to (10) and (17), the unified form of upper bound for the covariance, denoted by Ξ_{k+1}^i, is derived as

$$\begin{aligned} \Xi_{k+1}^i = \ & P_{k+1|k}^i - \gamma_{k+1}^i K_{k+1}^i P_{z_{k+1}z_{k+1}}^i (K_{k+1}^i)^T \\ & + (1-\gamma_{k+1}^i)\big[\eta_1 A_{k+1}^i P_{k+1|k}^i (A_{k+1}^i)^T \\ & + \eta_2 M_{k+1}^i R_{k+1}^i (M_{k+1}^i)^T + \eta_3 M_{k+1}^i \delta^i (M_{k+1}^i)^T \\ & - P_{k+1|k}^i\big]. \end{aligned} \tag{18}$$

Furthermore, the optimal filter gain M_{k+1}^i of node i when $\gamma_{k+1}^i = 0$, in the sense that it minimizes the upper bound $\bar{P}_{k+1}^i$, is derived via the following equation,

$$\begin{aligned} &\frac{\partial \mathrm{tr}(\bar{P}_{k+1}^i)}{\partial M_{k+1}^i} \\ &= \eta_1[-2(P_{k+1|k}^i)^T (H_{k+1}^i)^T] + \eta_1[2M_{k+1}^i H_{k+1}^i P_{k+1|k}^i \\ &\quad \times (H_{k+1}^i)^T] + \eta_2(2M_{k+1}^i R_{k+1}^i) + \eta_3(2M_{k+1}^i \delta^i) \\ &= 0. \end{aligned} \tag{19}$$

Accordingly, the filter gain M^i_{k+1} of node i can be written as

$$M^i_{k+1} = \eta_1 P^i_{k+1|k}(H^i_{k+1})^T\big[\eta_1 H^i_{k+1}P^i_{k+1|k}(H^i_{k+1})^T + \eta_2 R^i_{k+1} + \eta_3\delta^i I\big]^{-1}. \quad (20)$$

Remark 2. It is worth mentioning that the matrices P^i_{k+1} calculated by (16) are actually approximate error covariance matrices, which are not equal to the accurate covariance matrices in linear case due to the existence of linearization errors. Similar to Kluge et al. (2010), we denote them as error covariance matrices in this paper just for the sake of convenience. In addition, the upper bounds for the approximate error covariance matrices computed by (18) primarily serve as the cost functions to derive the filter gain matrices M^i_{k+1} when $\gamma^i_{k+1} = 0$.

Remark 3. It is clear that $M^i_{k+1}(M^i_{k+1})^T \geq 0$, thus the upper bound defined in (18) will increase with the increase of the triggering threshold δ^i. On the other hand, a larger threshold will result in less data transmission, which means that the thresholds δ^i do have a significant effect on the trade-off between estimation performance and communication rate.

Subsequently, we will develop the event-triggered cooperative UKF based on weighted average consensus algorithm such that all the local estimators can reach a consensus in terms of state estimates and upper bounds for the error covariances, denoted by the information pairs $(\hat{x}^i_{k+1}, \Xi^i_{k+1})$, $i \in \mathcal{V}$.

Definition 4 (*Li et al., 2016*)**.** The information pairs $(\hat{x}^i_{k+1}, \Xi^i_{k+1})$, $i \in \mathcal{V}$ are said to be of weighted average consensus, if the following limit exists for all $i \in \mathcal{V}$,

$$(\hat{x}^*_{k+1}, \Xi^*_{k+1}) = \lim_{l\to\infty}(\hat{x}^i_{k+1,l}, \Xi^i_{k+1,l}), \quad (21)$$

where l is the consensus step index and $(\hat{x}^i_{k+1,l}, \Xi^i_{k+1,l})$, $i \in \mathcal{V}$ denotes the information pair of node i available at time instant $k+1$ after lth iteration, satisfying

$$\begin{cases} \hat{x}^i_{k+1,l} = \sum_{j\in\mathcal{N}_i}\pi^{i,j}\hat{x}^j_{k+1,l-1} \\ \Xi^i_{k+1,l} = \sum_{j\in\mathcal{N}_i}\pi^{i,j}\Xi^j_{k+1,l-1} \end{cases} \quad (22)$$

with $\pi^{i,j} \geq 0$, being the weights and $\Sigma_{j\in\mathcal{N}_i}\pi^{i,j} = 1$. The initial conditions are $\hat{x}^i_{k+1,0} = \hat{x}^i_{k+1}$ and $\Xi^i_{k+1,0} = \Xi^i_{k+1}$.

Based on the above definition, the next theorem provides a sufficient condition for realizing a weighted average consensus with regard to the information pairs.

Theorem 5. *Consider the estimator network with communication topology $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. If the consensus weight matrix $\Pi = \{\pi^{i,j}\} \in \mathbb{R}^{n\times n}$ is chosen to be primitive, then each information pair $(\hat{x}^i_{k+1,l}, \Xi^i_{k+1,l})$, $i \in \mathcal{V}$ can achieve a weighted average consensus.*

Proof. The proof is similar to that of Theorem 1 in Li et al. (2016). The key in proof is based on the property of row-stochastic and primitive matrix Π (Ren, Beard, & Atkins, 2007), namely, there exists a column vector $\eta > 0$ with all its elements summing up to one, satisfying $\lim_{l\to\infty}\Pi^l = 1_N\eta^T$. Here, Π^l is the lth power of matrix Π. Interested readers can refer to Li et al. (2016) for more details.

Remark 6. It is worth pointing out that the connectivity of the graph is closely relevant to the primitivity of consensus weight matrix Π. For an undirected network, as stated in Calafiore and Abrate (2009), a consensus matrix is primitive provided that the graph is connected. On the other hand, for a directed network, the primitivity of consensus matrix can be guaranteed when the graph is strongly connected (Battistelli & Chisci, 2016).

Remark 7. The performance of consensus depends to a great extent on the number of consensus steps L at each sampling interval. As L increases, all elements of Π^L will gradually approach $\frac{1}{N}$. However, it is impossible to enlarge L without limits since both the transmission and calculation loads will increase as L increases. A proper L should be chosen to well balance the cost and consensus level (Battistelli et al., 2014). In fact, a small L can guarantee a desirable consensus performance if the undirected communication network is fully connected, or else a larger L is necessary (Li et al., 2016).

To conclude this section, the proposed event-triggered cooperative UKF algorithm is summarized as Algorithm 1 given below.

Remark 8. Note that the proposed cooperative filtering algorithm presents some significant differences when compared with the algorithms in Li et al. (2017), Xiong et al. (2006), in which only an independent sensor node is considered and the filtering is solely performed. However, in the proposed algorithm, each node can communicate with its neighboring peers and correct its own state estimation using the information broadcasted by its neighbors according to (22) with L iterations. Hence, the estimation performance can be improved to an extent through cooperation between nodes (see also Fig. 8 in Section 5).

4. Stability analysis of the proposed algorithm

In this section, the stochastic boundedness of filtering errors in mean square for the event-triggered cooperative UKF algorithm proposed in this paper will be analyzed.

Consider the discrete-time nonlinear system (1) and (2). For the convenience of analysis, the approach utilized in Xiong et al. (2006) and Li et al. (2017) is employed herein to simplify the error expressions, namely,

$$\tilde{x}^i_{k+1|k} = \alpha^i_k F^i_k \tilde{x}^i_k + \omega_k \quad (23)$$

$$\tilde{z}^i_{k+1} = \beta^i_{k+1}H^i_{k+1}\tilde{x}^i_{k+1|k} + \nu^i_{k+1}, \quad (24)$$

where $\tilde{z}^i_{k+1} = z^i_{k+1} - \hat{z}^i_{k+1|k}$ is the predicted measurement error. $F^i_k = \frac{\partial f(x_k)}{\partial x_k}\big|_{x_k=\hat{x}^i_k}$ and $H^i_{k+1} = \frac{\partial h^i(x_{k+1})}{\partial x_{k+1}}\big|_{x_{k+1}=\hat{x}^i_{k+1|k}}$ are Jacobian matrices. The unknown diagonal matrices $\alpha^i_k = \mathrm{diag}(\alpha^i_{k,1}, \alpha^i_{k,2}, \ldots, \alpha^i_{k,n})$ and $\beta^i_k = \mathrm{diag}(\beta^i_{k,1}, \beta^i_{k,2}, \ldots, \beta^i_{k,m})$ are introduced to compensate the linearization errors.

With (23) and (24) in hand, the predicted error covariance, predicted measurement covariance and state-measurement cross-covariance can be rearranged as

$$P^i_{k+1|k} = \alpha^i_k F^i_k \Xi^i_k(\alpha^i_k F^i_k)^T + Q_k \quad (25)$$

$$P^i_{z_{k+1}z_{k+1}} = \beta^i_{k+1}H^i_{k+1}P^i_{k+1|k}(H^i_{k+1})^T\beta^i_{k+1} + R^i_{k+1} \quad (26)$$

$$P^i_{x_{k+1}z_{k+1}} = P^i_{k+1|k}(H^i_{k+1})^T\beta^i_{k+1}. \quad (27)$$

In order to prove the stochastic boundedness of estimation error in mean square, it is necessary to introduce the following lemmas.

Lemma 9 (*Reif, Gunther, Yaz, & Unbehauen, 1999*)**.** *Assume there is a stochastic process $V_k(\xi_k)$ as well as real numbers $\underline{\varepsilon}, \overline{\varepsilon}, \mu > 0$ and $0 < \phi \leq 1$ such that*

$$\underline{\varepsilon}\|\xi_k\|^2 \leq V_k(\xi_k) \leq \overline{\varepsilon}\|\xi_k\|^2 \quad (28)$$

Algorithm 1: Event-triggered Cooperative Unscented Kalman Filtering.

Part A: Local estimation update for each node $i \in \mathcal{V}$

(1) Choose $2n+1$ sigma points at time instant k as

$$\chi_k^{i,s} = \hat{x}_k^i, \quad s = 0$$
$$\chi_k^{i,s} = \hat{x}_k^i + (\sqrt{(n+\lambda)\Xi_k^i})_s, \qquad s = 1, 2, \dots, n$$
$$\chi_k^{i,s} = \hat{x}_k^i - (\sqrt{(n+\lambda)\Xi_k^i})_{s-n}, \quad s = n+1, n+2, \dots, 2n.$$

(2) Calculate the predicted state and covariance

$$\chi_{k+1|k}^{i,s} = f(\chi_k^{i,s}), \quad s = 0, 1, 2, \dots, 2n$$
$$\hat{x}_{k+1|k}^i = \sum_{s=0}^{2n} W_s^m \chi_{k+1|k}^{i,s}$$
$$P_{k+1|k}^i = \sum_{s=0}^{2n} W_s^c (\chi_{k+1|k}^{i,s} - \hat{x}_{k+1|k}^i)(\chi_{k+1|k}^{i,s} - \hat{x}_{k+1|k}^i)^T + Q_k$$
$$\zeta_{k+1|k}^{i,s} = h(\chi_{k+1|k}^{i,s}), \quad s = 0, 1, 2, \dots, 2n$$
$$\hat{z}_{k+1|k}^i = \sum_{s=0}^{2n} W_s^m \zeta_{k+1|k}^{i,s}$$
$$P_{z_{k+1}z_{k+1}}^i = \sum_{s=0}^{2n} W_s^c (\zeta_{k+1|k}^{i,s} - \hat{z}_{k+1|k}^i)(\zeta_{k+1|k}^{i,s} - \hat{z}_{k+1|k}^i)^T + R_{k+1}^i$$
$$P_{x_{k+1}z_{k+1}}^i = \sum_{s=0}^{2n} W_s^c (\chi_{k+1|k}^{i,s} - \hat{x}_{k+1|k}^i)(\zeta_{k+1|k}^{i,s} - \hat{z}_{k+1|k}^i)^T.$$

where the scalar weights $W_0^m = \frac{\lambda}{n+\lambda}$, $W_0^c = \frac{\lambda}{n+\lambda} + 1 - \alpha^2 + \beta$, $W_s^m = W_s^c = \frac{1}{2(n+\lambda)}$, $s = 1, 2, \dots 2n$ and $\lambda = \alpha^2(n+\kappa) - n$.

(3) Calculate the value of γ_{k+1}^i using (3) and the filter gain matrices K_{k+1}^i or M_{k+1}^i through (9) and (20).

(4) Update the local estimates $\hat{x}_{k+1}^i$ and Ξ_{k+1}^i for each node $i \in \mathcal{V}$ through (6) and (18).

Part B: Cooperative update based on neighbors

(5) Initialize the consensus algorithm as $\hat{x}_{k+1,0}^i = \hat{x}_{k+1}^i$, $\Xi_{k+1,0}^i = \Xi_{k+1}^i$.

(6) Communicate the information pair $(\hat{x}_{k+1}^i, \Xi_{k+1}^i)$ with neighborhoods.

(7) Fuse the information according to (22).

(8) Update the estimates after L iterations as $\hat{x}_{k+1}^i = \hat{x}_{k+1,L}^i$, $\Xi_{k+1}^i = \Xi_{k+1,L}^i$.

(9) Repeat the above steps.

and

$$\mathbb{E}\{V_k(\xi_k)|\xi_{k-1}\} - V_{k-1}(\xi_{k-1}) \le \mu - \phi V_{k-1}(\xi_{k-1}) \tag{29}$$

are fulfilled. Then the stochastic process is exponentially bounded in mean square, i.e., we have

$$\mathbb{E}\{\|\xi_k\|^2\} \le \frac{\overline{\varepsilon}}{\underline{\varepsilon}}\mathbb{E}\{\|\xi_0\|^2\}(1-\phi)^k + \frac{\mu}{\underline{\varepsilon}}\sum_{i=1}^{k-1}(1-\phi)^i \tag{30}$$

and the stochastic process is bounded with probability one.

Lemma 10 (Xiong et al., 2006). *Assume that matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$, if $A > 0$ and $B > 0$, then*

$$A^{-1} > (A+B)^{-1}. \tag{31}$$

Lemma 11 (Xiong et al., 2006). *Assume that matrices $A \in \mathbb{R}^{m\times m}$, $B \in \mathbb{R}^{m\times n}$, $C \in \mathbb{R}^{n\times n}$, if $A > 0$ and $C > 0$, then*

$$A^{-1} > B(B^TAB + C)^{-1}B^T. \tag{32}$$

Lemma 12 (Battistelli & Chisci, 2014). *Given an integer $N \ge 2$, a set of positive definite matrices $\{M_i\}_{i=1}^N$ and a set of vectors $\{\nu_i\}_{i=1}^N$, the following inequality holds*

$$\Big(\sum_{i=1}^N M_i\nu_i\Big)^T\Big(\sum_{i=1}^N M_i\Big)^{-1}\Big(\sum_{i=1}^N M_i\nu_i\Big) \le \sum_{i=1}^N \nu_i^T M_i \nu_i. \tag{33}$$

Lemma 13 (Matei & Baras, 2012). *Given a positive integer N, a set of vectors $\{\nu_i\}_{i=1}^N$, a set of non-negative scalars $\{p_i\}_{i=1}^N$ summing up to one, and a positive definite matrix Q, the following inequality holds*

$$\Big(\sum_{i=1}^N p_i\nu_i\Big)^T Q \Big(\sum_{i=1}^N p_i\nu_i\Big) \le \sum_{i=1}^N p_i\nu_i^T Q \nu_i. \tag{34}$$

With these aforementioned lemmas and formulation as well as Algorithm 1, it is ready to state and prove the main result of this paper.

Theorem 14. *Consider a sensor network described by the nonlinear stochastic systems* (1) and (2), *as well as Algorithm 1. The estimation error $\tilde{x}_{k+1}^i = x_{k+1} - \hat{x}_{k+1}^i$ is exponentially bounded in mean square for any $i \in \mathcal{V}$ providing that the following assumptions are satisfied.*
(1) Real numbers $\underline{\alpha}, \underline{f}, \underline{\beta}, \underline{h} \neq 0$ and $\overline{\alpha}, \overline{f}, \overline{\beta}, \overline{h} \neq 0$ exist such that the following inequalities always hold:

$$\begin{cases} \underline{\alpha}^2 I_n \le \alpha_k^i(\alpha_k^i)^T \le \overline{\alpha}^2 I_n, \ \underline{f}^2 I_n \le F_k^i(F_k^i)^T \le \overline{f}^2 I_n \\ \underline{\beta}^2 I_m \le \beta_k^i(\beta_k^i)^T \le \overline{\beta}^2 I_m, \ \underline{h}^2 I_m \le H_k^i(H_k^i)^T \le \overline{h}^2 I_m. \end{cases} \tag{35}$$

(2) Real numbers $p_{max} \ge p_{min} > 0$, $\overline{q} \ge \underline{q} > 0$, $\overline{r} \ge \underline{r} > 0$ and $\overline{p} \ge \underline{p} > 0$ exist such that the following inequalities always hold:

$$\begin{cases} p_{\min} \le p^i \le p_{\max}, \quad \underline{q} I_n \le Q_k \le \overline{q} I_n \\ \underline{r} I_m \le R_k^i \le \overline{r} I_m, \ \underline{p} I_n \le P_{k+1|k}^i \le \overline{p} I_n. \end{cases} \tag{36}$$

(3) The consensus weight matrix Π is row-stochastic and primitive.

Proof. At first, let us denote $\tilde{x}_{k+1|k} = \mathrm{col}(\tilde{x}_{k+1|k}^i, i \in \mathcal{V})$ and $\tilde{x}_k = \mathrm{col}(\tilde{x}_k^i, i \in \mathcal{V})$. Denote $p = (p^1, \dots, p^i, \dots, p^N)^T$ as the Perron–Frobenius left eigenvector of the matrix Π^L, where $\Pi^L = (\pi_L^{i,j})_{n\times n}$. According to Assumption (3), it can be noted that p^i is a strictly positive component and $p^T\Pi^L = p^T$, namely, $\sum_{j\in\mathcal{V}} p^j\pi_L^{j,i} = p^i$.

Define the following stochastic process with respect to $\tilde{x}_{k+1|k}$:

$$V_{k+1}(\tilde{x}_{k+1|k}) = \sum_{i\in\mathcal{V}} p^i(\tilde{x}_{k+1|k}^i)^T(P_{k+1|k}^i)^{-1}\tilde{x}_{k+1|k}^i. \tag{37}$$

According to the assumption that $\underline{p}I_n \le P_{k+1|k}^i \le \overline{p}I_n$, which can be derived by Theorem 2 in Kluge et al. (2010), we easily get

$$\frac{p_{\min}}{\overline{p}}\|\tilde{x}_{k+1|k}\|^2 \le V_{k+1}(\tilde{x}_{k+1|k}) \le \frac{p_{\max}}{\underline{p}}\|\tilde{x}_{k+1|k}\|^2, \tag{38}$$

which meets the condition (28) of Lemma 9. Note that $\sum_{j\in\mathcal{V}}\pi_L^{i,j} = 1$ by consensus, then it follows from (6), (15), (23) and (24) that

$$\begin{aligned} &\tilde{x}_{k+1|k}^i \\ =\ &\alpha_k^i F_k^i(x_k - \hat{x}_k^i) + \omega_k \\ =\ &\alpha_k^i F_k^i\Big[\sum_{j\in\mathcal{V}}\pi_L^{i,j}(x_k - \hat{x}_{k,0}^j)\Big] + \omega_k \\ =\ &\alpha_k^i F_k^i\Big[\sum_{j\in\mathcal{V}}\pi_L^{i,j}(x_k - \hat{x}_{k|k-1}^j - \gamma_k^j K_k^j(z_k^j - \hat{z}_{k|k-1}^j)\end{aligned}$$

$$-(1-\gamma_k^j)M_k^j(z^j_{\tau^j_{k-1}}-\hat{z}^j_{k|k-1}))\big]+\omega_k$$
$$=\sum_{j\in\mathcal{V}}\Omega_k^{i,j}\tilde{x}^j_{k|k-1}+\sum_{j\in\mathcal{V}}\Lambda_k^{i,j}(z_k^j-z^j_{\tau^j_{k-1}})$$
$$+\sum_{j\in\mathcal{V}}\Theta_k^{i,j}v_k^j+\omega_k \tag{39}$$

where

$$\begin{cases}\Omega_k^{i,j}=\pi_L^{i,j}\alpha_k^iF_k^i\big(I-\gamma_k^jK_k^j\beta_k^jH_k^j-(1-\gamma_k^j)M_k^jH_k^j\big)\\ \Lambda_k^{i,j}=\pi_L^{i,j}\alpha_k^iF_k^i(1-\gamma_k^j)M_k^j\\ \Theta_k^{i,j}=\pi_L^{i,j}\alpha_k^iF_k^i\big(-\gamma_k^jK_k^j-(1-\gamma_k^j)M_k^j\big).\end{cases}$$

As a matter of convenience, let us define $\mathcal{R}_k^{i,j}=\sum_{j\in\mathcal{V}}\Omega_k^{i,j}\tilde{x}^j_{k|k-1}$, $\mathcal{S}_k^{i,j}=\sum_{j\in\mathcal{V}}\Lambda_k^{i,j}(z_k^j-z^j_{\tau^j_{k-1}})$, $\mathcal{T}_k^{i,j}=\mathcal{U}_k^{i,j}+\omega_k$, $\mathcal{U}_k^{i,j}=\sum_{j\in\mathcal{V}}\Theta_k^{i,j}v_k^j$.

Next, substituting (39) to (37) and taking the conditional expectation yields

$$\mathbb{E}\left\{V_{k+1}(\tilde{x}_{k+1|k})|\tilde{x}_{k|k-1}\right\}$$
$$=\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i(\tilde{x}^i_{k+1|k})^T(P^i_{k+1|k})^{-1}\tilde{x}^i_{k+1|k}|\tilde{x}_{k|k-1}\Big\}$$
$$=\Psi^x_{k+1}+\Psi^z_{k+1}+\Psi^{v,\omega}_{k+1}+\Psi^{x,z}_{k+1}+\Psi^{z,v,\omega}_{k+1} \tag{40}$$

where

$$\Psi^x_{k+1}=\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i(\mathcal{R}_k^{i,j})^T(P^i_{k+1|k})^{-1}\mathcal{R}_k^{i,j}|\tilde{x}_{k|k-1}\Big\},$$
$$\Psi^z_{k+1}=\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i(\mathcal{S}_k^{i,j})^T(P^i_{k+1|k})^{-1}\mathcal{S}_k^{i,j}|\tilde{x}_{k|k-1}\Big\},$$
$$\Psi^{v,\omega}_{k+1}=\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i(\mathcal{T}_k^{i,j})^T(P^i_{k+1|k})^{-1}\mathcal{T}_k^{i,j}|\tilde{x}_{k|k-1}\Big\},$$
$$\Psi^{x,z}_{k+1}=\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i\Big[(\mathcal{R}_k^{i,j})^T(P^i_{k+1|k})^{-1}\mathcal{S}_k^{i,j}+(\mathcal{S}_k^{i,j})^T(P^i_{k+1|k})^{-1}\mathcal{R}_k^{i,j}\Big]|\tilde{x}_{k|k-1}\Big\},$$
$$\Psi^{z,v,\omega}_{k+1}=\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i\Big[(\mathcal{S}_k^{i,j})^T(P^i_{k+1|k})^{-1}\mathcal{T}_k^{i,j}+(\mathcal{T}_k^{i,j})^T(P^i_{k+1|k})^{-1}\mathcal{S}_k^{i,j}\Big]|\tilde{x}_{k|k-1}\Big\}.$$

In what follows, we will discuss each of these five terms in (40). Firstly, we focus on the first term Ψ^x_{k+1}, recall (25) and apply Lemma 10, it is immediate to see that

$$(P^i_{k+1|k})^{-1}\le(\alpha_k^iF_k^i)^{-T}(\Xi_k^i)^{-1}(\alpha_k^iF_k^i)^{-1}, \tag{41}$$

the non-singularity of $\alpha_k^iF_k^i$ can be guaranteed by Assumption (1) of Theorem 14. Substituting (41) into the expression of Ψ^x_{k+1} gives

$$\Psi^x_{k+1}\le\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i(\sum_{j\in\mathcal{V}}\pi_L^{i,j}\Gamma_k^j\tilde{x}^j_{k|k-1})^T(\Xi_k^i)^{-1}\times(\sum_{j\in\mathcal{V}}\pi_L^{i,j}\Gamma_k^j\tilde{x}^j_{k|k-1})|\tilde{x}_{k|k-1}\Big\} \tag{42}$$

where $\Gamma_k^j=I-\gamma_k^jK_k^j\beta_k^jH_k^j-(1-\gamma_k^j)M_k^jH_k^j$ for convenience.

According to (9), (18), (20), (26) and (27) as well as the consensus algorithm, we have

$$\Xi_k^i=\sum_{j\in\mathcal{V}}\pi_L^{i,j}\Xi^j_{k,0}\ge\sum_{j\in\mathcal{V}}\pi_L^{i,j}\Gamma_k^jP^j_{k|k-1}. \tag{43}$$

Thus, we can rewrite (42) as

$$\Psi^x_{k+1}$$
$$\le\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i(\sum_{j\in\mathcal{V}}\pi_L^{i,j}\Gamma_k^jP^j_{k|k-1}(P^j_{k|k-1})^{-1}\tilde{x}^j_{k|k-1})^T$$
$$\times(\sum_{j\in\mathcal{V}}\pi_L^{i,j}\Gamma_k^jP^j_{k|k-1})^{-1}(\sum_{j\in\mathcal{V}}\pi_L^{i,j}\Gamma_k^jP^j_{k|k-1}$$
$$\times(P^j_{k|k-1})^{-1}\tilde{x}^j_{k|k-1})|\tilde{x}_{k|k-1}\Big\}. \tag{44}$$

Applying Lemma 12 to (44) and noting that $\sum_{j\in\mathcal{V}}p^j\pi_L^{j,i}=p^i$ yields

$$\Psi^x_{k+1}$$
$$\le\mathbb{E}\Big\{\sum_{i\in\mathcal{V}}p^i\sum_{j\in\mathcal{V}}\pi_L^{i,j}(\tilde{x}^j_{k|k-1})^T(P^j_{k|k-1})^{-1}\times(I-\gamma_k^jK_k^j\beta_k^jH_k^j-(1-\gamma_k^j)M_k^jH_k^j)\tilde{x}^j_{k|k-1}|\tilde{x}_{k|k-1}\Big\}$$
$$=\mathbb{E}\Big\{\sum_{j\in\mathcal{V}}p^j(\tilde{x}^j_{k|k-1})^T(P^j_{k|k-1})^{-1}\tilde{x}^j_{k|k-1}|\tilde{x}_{k|k-1}\Big\}$$
$$-\mathbb{E}\Big\{\sum_{j\in\mathcal{V}}p^j(\tilde{x}^j_{k|k-1})^T(P^j_{k|k-1})^{-1}\times(\gamma_k^jK_k^j\beta_k^jH_k^j+(1-\gamma_k^j)M_k^jH_k^j)\tilde{x}^j_{k|k-1}|\tilde{x}_{k|k-1}\Big\}. \tag{45}$$

Next, we discuss the second half part of (45) in two cases. If $\gamma_k^j=1$, inserting (26) and (27) into (9), and substituting the rearranged expression of K_k^j into (45), then on the basis of Lemma 11, we can find a real number $0<\phi<\phi_1\le 1$, where

$$\phi_1=\min\big\{\lambda_{\min}\{(\beta_s^jH_s^j)^T[\beta_s^jH_s^jP^j_{s|s-1}(\beta_s^jH_s^j)^T+R_s^j]^{-1}\beta_s^jH_s^j\}/\lambda_{\max}[(P^j_{s|s-1})^{-1}],j\in V,s=0,1,\ldots,k\big\},$$

such that

$$\phi(P^j_{k|k-1})^{-1}$$
$$\le(\beta_k^jH_k^j)^T[\beta_k^jH_k^jP^j_{k|k-1}(\beta_k^jH_k^j)^T+R_k^j]^{-1}\beta_k^jH_k^j, \tag{46}$$

which is consistent with the results in Li et al. (2016). If $\gamma_k^j=0$, substituting (20) into (45), similarly, we can also find a real number $0<\phi<\phi_2\le 1$, where

$$\phi_2=\min\big\{\lambda_{\min}\{(H_s^j)^T[\eta_1H_s^jP^j_{s|s-1}(H_s^j)^T+\eta_2R_s^j+\eta_3\delta^jI]^{-1}H_s^j\}/\lambda_{\max}\{[(1+\sigma_1)P^j_{s|s-1}]^{-1}\},$$
$$j\in V,s=0,1,\ldots,k\big\},$$

such that

$$\phi[\eta_1P^j_{k|k-1}]^{-1}$$
$$\le(H_k^j)^T[\eta_1H_k^jP^j_{k|k-1}(H_k^j)^T+\eta_2R_k^j+\eta_3\delta^jI]^{-1}H_k^j. \tag{47}$$

Define $\phi^*=\min\{\phi_1,\phi_2\}$, let $0<\phi<\phi^*\le 1$, (45) can be rewritten as

$$\Psi^x_{k+1}\le(1-\phi)\mathbb{E}\{V_k(\tilde{x}_{k|k-1})\}. \tag{48}$$

Now, we proceed with the second term Ψ^z_{k+1}, by virtue of Lemma 13 and $(1-\gamma_k^j)^2=1-\gamma_k^j$, we have

$$\Psi^z_{k+1}\le\frac{\overline{\alpha}^2\overline{f}^2\overline{m}^2}{\underline{p}}\mathbb{E}\{\sum_{i\in\mathcal{V}}p^i\sum_{j\in\mathcal{V}}\pi_L^{i,j}(1-\gamma_k^j)\times(z_k^j-z^j_{\tau^j_{k-1}})^T(z_k^j-z^j_{\tau^j_{k-1}})|\tilde{x}_{k|k-1}\}$$
$$\le\frac{\overline{\alpha}^2\overline{f}^2\overline{m}^2}{\underline{p}}\sum_{j\in\mathcal{V}}p^j\delta^j. \tag{49}$$

where we define $\overline{m}=\frac{\eta_1\overline{p}\overline{h}}{\eta_1\underline{p}\underline{h}^2+\eta_2\underline{r}+\eta_3\underline{\delta}}$, $\underline{\delta}$ is the minimum trigger threshold.

Consider the third term $\Psi^{\nu,\omega}_{k+1}$,

$$\begin{aligned}&\Psi^{\nu,\omega}_{k+1}\\ \leq\ &\frac{1}{\underline{p}}\mathbb{E}\{\sum_{i\in\mathcal{V}}p^i[\mathrm{tr}\{(\mathcal{U}^{i,j}_k)^T\mathcal{U}^{i,j}_k\}\\ &+\mathrm{tr}\{\omega_k^T\omega_k\}]|\tilde{x}_{k|k-1}\}\\ \leq\ &\frac{1}{\underline{p}}\{\sum_{i\in\mathcal{V}}p^i[\overline{\alpha}^2\overline{f}^2\overline{r}(\overline{k}^2+\overline{m}^2)n\sum_{j\in\mathcal{V}}\pi_L^{i,j}+\overline{q}n]\}\\ =\ &\frac{(\overline{\alpha}^2\overline{f}^2\overline{r}(\overline{k}^2+\overline{m}^2)+\overline{q})n}{\underline{p}}\sum_{j\in\mathcal{V}}p^j. \end{aligned}\tag{50}$$

where we define $\overline{k}=\frac{\overline{p}\overline{\beta}\overline{h}}{\underline{p}\underline{\beta}^2\underline{h}^2+\underline{r}}$.

Next, analyzing the fourth term $\Psi^{x,z}_{k+1}$ and applying Lemmas 1 and 13, we obtain

$$\begin{aligned}\Psi^{x,z}_{k+1}\leq\ &\frac{1}{\underline{p}}\mathbb{E}\{\sum_{i\in\mathcal{V}}p^i\mathrm{tr}[\sigma_3\mathcal{R}^{i,j}_k(\mathcal{R}^{i,j}_k)^T\\ &+\sigma_3^{-1}\mathcal{S}^{i,j}_k(\mathcal{S}^{i,j}_k)^T]|\tilde{x}_{k|k-1}\}\\ \leq\ &\frac{\sigma_3\overline{p}\overline{\alpha}^2\overline{f}^2(1+\overline{k}\overline{\beta}\overline{h}+\overline{m}\overline{h})^2n}{\underline{p}}\sum_{j\in\mathcal{V}}p^j\\ &+\frac{\sigma_3^{-1}\overline{\alpha}^2\overline{f}^2\overline{m}^2}{\underline{p}}\sum_{j\in\mathcal{V}}p^j\delta^j.\end{aligned}\tag{51}$$

Similarly, we consider the last term $\Psi^{z,\nu,\omega}_{k+1}$,

$$\begin{aligned}\Psi^{z,\nu,\omega}_{k+1}\leq\ &\frac{\sigma_4(\overline{\alpha}^2\overline{f}^2\overline{r}(\overline{k}^2+\overline{m}^2)+\overline{q})n}{\underline{p}}\sum_{j\in\mathcal{V}}p^j\\ &+\frac{\sigma_4^{-1}\overline{\alpha}^2\overline{f}^2\overline{m}^2}{\underline{p}}\sum_{j\in\mathcal{V}}p^j\delta^j\end{aligned}\tag{52}$$

where $\sigma_3>0$ and $\sigma_4>0$ are scalars.

Now, we define

$$\begin{aligned}\mu\triangleq\ &\frac{(1+\sigma_4)(\overline{\alpha}^2\overline{f}^2\overline{r}(\overline{k}^2+\overline{m}^2)+\overline{q})n+\sigma_3\overline{p}\overline{\alpha}^2\overline{f}^2(1+\overline{k}\overline{\beta}\overline{h}+\overline{m}\overline{h})^2n}{\underline{p}}\\ &\times\sum_{j\in\mathcal{V}}p^j+\frac{(1+\sigma_3^{-1}+\sigma_4^{-1})\overline{\alpha}^2\overline{f}^2\overline{m}^2}{\underline{p}}\sum_{j\in\mathcal{V}}p^j\delta^j.\end{aligned}$$

Then, condition (29) is satisfied. According to Lemma 9, $\tilde{x}_{k+1|k}$ is exponentially bounded in mean square, which naturally indicates that each component $\tilde{x}^i_{k+1|k}$ is also exponentially bounded in mean square.

In order to prove the stochastic boundedness of the estimation error $\tilde{x}^i_{k+1}$ in mean square, taking expectation from both sides of (23), we can see that

$$\mathbb{E}\{\|\tilde{x}^i_k\|^2\}\leq\underline{\alpha}^{-2}\underline{f}^{-2}(\mathbb{E}\{\|\tilde{x}^i_{k+1|k}\|^2\}-\mathbb{E}\{\|\omega_k\|^2\}).\tag{53}$$

Since the process noise ω_k is also exponentially bounded in mean square by applying the same method, we can naturally draw a conclusion that the estimation error $\tilde{x}^i_{k+1}$ is exponentially bounded in mean square, which completes the proof.

Remark 15. Assumption (1) has been widely used for the stability analysis of the single Kalman-like filter (Kluge et al., 2010; Li & Xia, 2012; Reif et al., 1999) and the consensus-based one (Battistelli & Chisci, 2016; Li et al., 2016). The bounds on F^i_k and H^i_k will hold provided that the functions f and h^i are globally Lipschitz (Li, Jia, & Du, 2018). Assumption (2) means the boundedness of the covariance matrices, which is common in many works discussing the stochastic stability (Li et al., 2017; Xiong et al., 2006; Zheng & Fang, 2016). As for Assumption (3), the Metropolis weights (Xiao, Boyd, & Lall, 2005) can guarantee, without a doubt, that the consensus weight matrix Π is row-stochastic. Further, the primitivity is associated with the connectivity of the graph (see also Remark 6).

Remark 16. Note that, the utilization of Lemma 1 when deriving an upper bound Ξ^i_{k+1} for the filtering error covariance may bring in conservativeness. Such conservativeness can be lessened by selecting apposite scaling parameters σ_1 and σ_2. As can be seen from the equations related with σ_1 and σ_2 in the above stability analysis, the boundedness of the estimation errors for the proposed algorithm can always be guaranteed with different values of σ_1 and σ_2, even though the values of ϕ and μ will vary accordingly.

5. Numerical results and analysis

This section aims to demonstrate the theoretical results in previous sections via analyzing the effectiveness of the developed algorithm. A practical scenario involving a non-cooperative moving target localization using multiple UAVs is utilized here to justify the potential applicability of the proposed event-triggered distributed filtering scheme.

As stated in Zengin and Dogan (2006), pursuing highly maneuvering targets is regarded as one of the representative applications of multiple UAVs with onboard sensors, especially in an adversarial environment. In a typical military application, the operators in the ground station (command center) can command a group of UAVs equipped with battery-powered sensors and event-triggered schedulers to follow the non-cooperative maneuvering target. Assume that the corresponding estimator of each UAV is placed in the ground station. To reduce the communication costs in sensor-to-estimator channels and extend the working hours of UAVs, measurement signals are transmitted to the corresponding estimators only when the pre-specified triggering conditions are satisfied. For simplicity, the relative distance and angle information from the target to each UAV are assumed to be available as long as the moving target locates within the detectable range of UAVs. It is worth mentioning that a detailed discussion about the cooperative control problem of multiple UAVs is beyond the scope of this paper and hence we refer the interested readers to the related work (Ma & Hovakimyan, 2013) and references therein. The overall estimation architecture is depicted in Fig. 1. It can be seen that each ground estimator associated with its own UAV can exchange the local information with its neighbors. That is the main reason why the multiple UAVs can work cooperatively to track the ground moving target. The kinematic model for the ground moving target is written as

$$\begin{aligned}x^t_{k+1}&=x^t_k+v_k cos(\theta_k)+\omega^x_k\\ y^t_{k+1}&=y^t_k+v_k sin(\theta_k)+\omega^y_k\\ v_{k+1}&=v_k+\varphi_k+\omega^v_k\\ \theta_{k+1}&=\theta_k+\phi_k+\omega^\theta_k\end{aligned}$$

where (x^t_k, y^t_k) denotes the position and v_k, θ_k, φ_k, ϕ_k are velocity, heading, acceleration and turn rate, respectively. Let $\omega_k=(\omega^x_k,\omega^y_k,\omega^v_k,\omega^\theta_k)$ be zero mean Gaussian white noise sequence with covariance Q_k.

In the multiple UAVs tracking system, four UAVs are utilized to detect the ground moving target and the measurement model

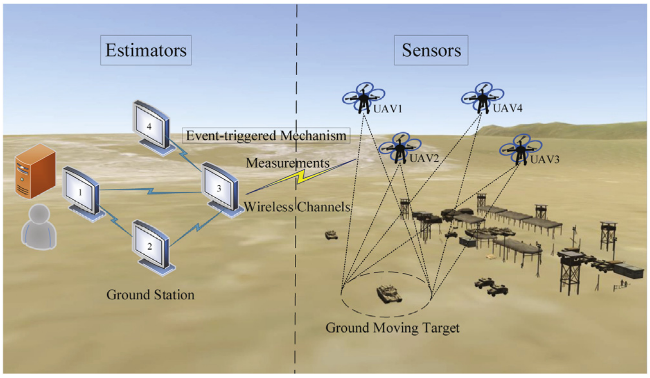

Fig. 1. Architecture of the moving target localization using multiple UAVs tracking system.

is simply described by Crassidis and Junkins (2004)

$$z_k^i = \begin{bmatrix} \sqrt{(x_k^{u,i}-x_k^t)^2+(y_k^{u,i}-y_k^t)^2+(z_k^{u,i})^2}+v_k^{d,i} \\ \arctan\left(\dfrac{z_k^{u,i}}{\sqrt{(x_k^{u,i}-x_k^t)^2+(y_k^{u,i}-y_k^t)^2}}\right)+v_k^{e,i} \\ \arctan\left(\dfrac{y_k^{u,i}-y_k^t}{x_k^{u,i}-x_k^t}\right)+v_k^{a,i} \end{bmatrix}, \quad i=1,2,3,4$$

where $(x_k^{u,i}, y_k^{u,i}, z_k^{u,i})$ denotes the position of the ith UAV, which is directly obtained from onboard GPS. $v_k^i = (v_k^{d,i}, v_k^{e,i}, v_k^{a,i})$ is zero mean Gaussian white measurement noise with covariance R_k^i. The communication network among ground estimators shown in Fig. 1 is described by a connected undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with nodes set $\mathcal{V} = \{1, 2, 3, 4\}$. The consensus weights are set equal to Metropolis weights (Xiao et al., 2005), i.e.

$$\pi^{i,j} = \begin{cases} 1/(1+\max\{d_i, d_j\}), & \text{if } (i,j)\in\mathcal{E} \\ 1-\sum_{(i,j)\in\mathcal{E}}\pi^{i,j}, & \text{if } i=j \\ 0, & \text{otherwise.} \end{cases}$$

In the simulations, the parameters are chosen as $\alpha = 10^{-2}$, $\beta = 2$, $n+\kappa = 3$, $\sigma_1 = \sigma_2 = 0.02$, $L = 5$, $\varphi_k = 0.1$, $\phi_k = 0.1$, $Q_k = \mathrm{diag}\{0.1, 0.1, 0.001, 0.001\}$, $R_k^1 = R_k^2 = R_k^3 = R_k^4 = \mathrm{diag}\{0.1, 0.001, 0.001\}$. Set the initial values as $x_{0|0} = [10, 10, 2, \pi/6]^T$, $\hat{x}_{0|0}^1 = [10.5, 10.5, 1.8, 0.7]^T$, $\hat{x}_{0|0}^2 = [11, 11, 2.3, 0.6]^T$, $\hat{x}_{0|0}^3 = [9.5, 9.5, 2.2, 0.8]^T$, $\hat{x}_{0|0}^4 = [9, 9, 1.7, 0.3]^T$, and $P_{0|0}^1 = P_{0|0}^2 = P_{0|0}^3 = P_{0|0}^4 = \mathrm{diag}\{1, 1, 0.1, 0.1\}$.

The averaged root mean square errors (RMSEs) are used herein to evaluate the tracking performance of the algorithms. The RMSEs of the position estimates over N_m Monte Carlo runs are defined by

$$\mathrm{RMSE}(k) = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\frac{1}{N_m}\sum_{m=1}^{N_m}\left((x_k^t-\hat{x}_{k|k}^{t,i})^2+(y_k^t-\hat{y}_{k|k}^{t,i})^2\right)}.$$

Moreover, the average communication rate for networks is similarly defined as Li et al. (2017), i.e. $\overline{\gamma} = \frac{1}{N}\sum_{i=1}^{N}\overline{\gamma}^i$, where $\overline{\gamma}^i = \frac{1}{K}\sum_{k=1}^{K}\gamma_k^i$ denotes the average communication rate for node i and K is the number of samples. Without loss of generality, all the triggering thresholds are assumed to be identical in this simulation (i.e. $\delta^1 = \delta^2 = \delta^3 = \delta^4 = \delta$). The behaviors of actual states and their respective estimations with triggering threshold $\delta = 1$ are depicted in Figs. 2–6, which illustrates that the presented algorithm has a satisfactory estimation performance. Compared with the time-triggered mechanism, the measurement transmission times are significantly reduced in Fig. 7.

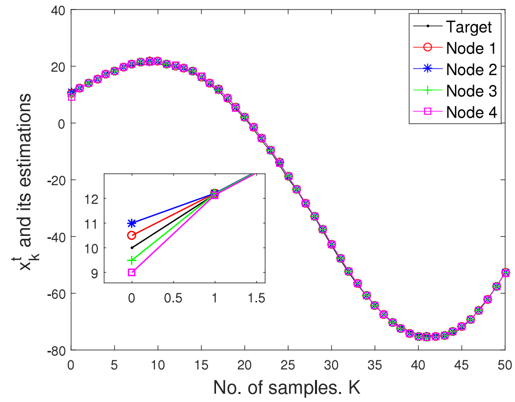

Fig. 2. Actual and estimated states of x_k^t.

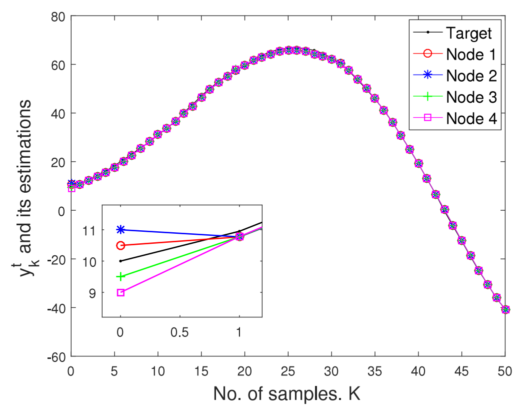

Fig. 3. Actual and estimated states of y_k^t.

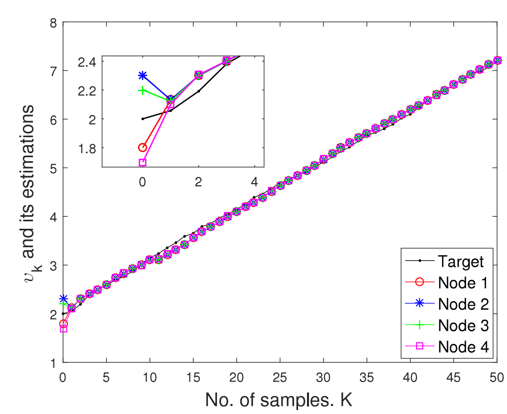

Fig. 4. Actual and estimated states of v_k.

To verify the necessity of using multiple UAVs to cooperatively track a ground target, the proposed event-triggered cooperative UKF (denoted by ECUKF) is compared with the standard UKF, where only an UAV is utilized. Then the ECUKF algorithm is further tested with different triggering thresholds. In fact, the ECUKF degenerates to the CUKF in Li et al. (2016) if the event-triggered threshold is taken to be $\delta = 0$. The behaviors of the RMSEs obtained over 150 Monte Carlo runs are shown in Fig. 8. As can be seen, the ECUKF algorithm is roughly the same as the UKF algorithm when $\delta = 0.1$, despite the event-triggered data

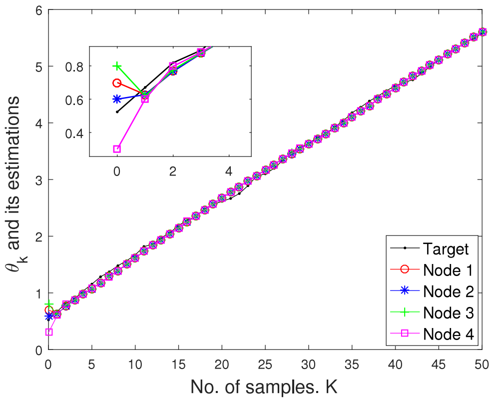

Fig. 5. Actual and estimated states of θ_k.

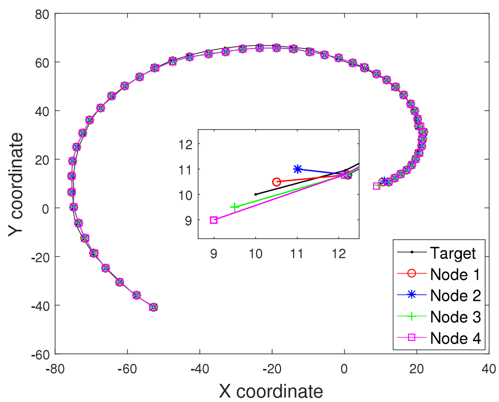

Fig. 6. Actual and estimated trajectories of the ground moving target.

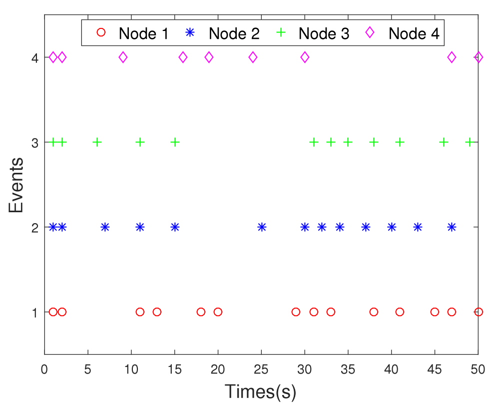

Fig. 7. Event-triggered times for each node.

transmission mechanism leads to a reduction in the number of measurement transmission. This is exactly how the consensus algorithm compensates the effect of event-triggered transmission to a great extent. Besides, the ECUKF suffers more performance degradation as the triggering threshold increases. To this end, the influence of triggering thresholds on the average communication rate for networks is shown in Table 1, from which we can see that the increase of triggering thresholds would contribute to a lower measurement transmission frequency.

6. Conclusions

In this paper, we have investigated the event-triggered communication schedules in weighted average consensus-based UKF framework. A significant reduction of the average sensor-to-estimator communication rate can be realized on basis of the event-triggered communication scheme. Furthermore, we can get a desired balance between filtering performance and communication rate by properly adjusting the triggering threshold. In addition, the guaranteed stability of the proposed algorithm is proved by means of stochastic stability theory. Finally, the proposed algorithm is applied to non-cooperative moving target localization with multiple UAVs, which demonstrates the effectiveness of the proposed algorithm. It can be noted from the simulation results that the upper bounds for the approximate error covariance matrices can work well, but more analysis should be done in the future to further understand the real impact on the estimation performance. Future work would also focus on designing an adaptive filtering algorithm with time-varying threshold to obtain the desired data transmission rate and considering the event-triggered mechanism in estimator-to-estimator channel as well to further relax the communication burdens.

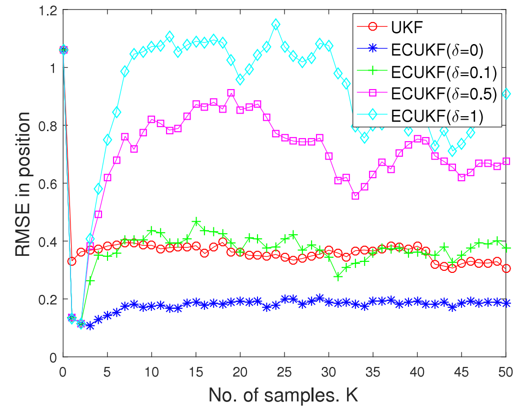

Fig. 8. Performance comparison with respect to RMSE in position.

Table 1
Influence of triggering thresholds.

δ	0	0.1	0.5	1
$\overline{\gamma}$	1.0000	0.5950	0.3150	0.2400

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