Bearing Rigidity Theory and its Applications for Control and Estimation of Network Systems

Shiyu Zhao

Department of Automatic Control and Systems Engineering
University of Sheffield, UK

Nanyang Technological University, Jan 2018

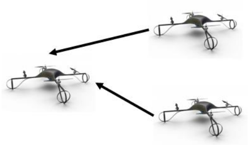
My research interests

Networked unmanned aerial vehicle (UAV) systems

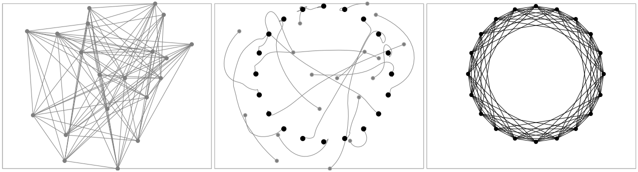
- Low-level: guidance, navigation, and flight control of single UAVs
- High-level: air traffic control, distributed control and estimation over multiple UAVs
- Application of vision sensing: vision-based guidance, navigation, and coordination control

Research motivation

Vision-based formation control of UAVs

Two problems: formation control and vision sensing
1) formation control:



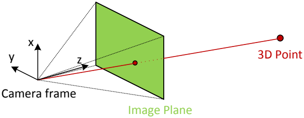
(a) Initial formation (b) trajectory (c) Final

Mature, require relative position measurements

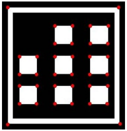
Research motivation

2) vision sensing

Step 1: recognition and tracking



Step 2: position estimation from bearings



Challenge: it is difficult to obtain accurate distance or relative position

Research motivation

◇ Idea: formation control merely using bearing-only measurements

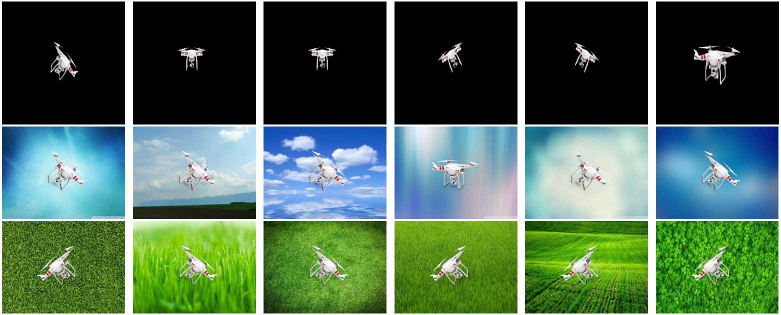
◇ Advantages: In practice, reduce the complexity of vision system. In theory, prove that distance information is redundant.

◇ Challenges:

- Nonlinear system (linear if position feedback is available)
- A relatively new topic that had not been studied

◇ The focus of this talk: bearing-only formation control and related topics

◇ Vision sensing: ongoing research



Outline

1 Bearing rigidity theory

2 Bearing-only formation control

3 Bearing-based network localization

4 Bearing-based formation control

5 Formation control with motion constraints

6 Affine formation maneuver control

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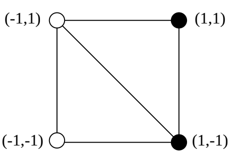
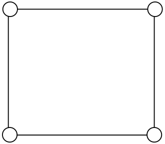
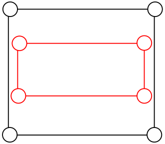
With bearing feedback, we control inter-agent bearings



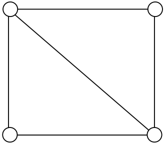
Question: when bearings can determine a unique formation shape?



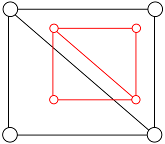
(a) Desired formation



(b) Resut



(a) Desired formation



(b) Result

◇ Notations:

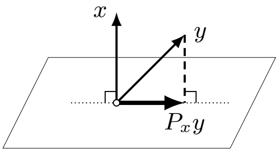
- Graph: $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, \dots, n\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$
- Configuration: $p_i \in \mathbb{R}^d$ with $i \in \mathcal{V}$ and $p = [p_1^{\mathrm{T}}, \dots, p_n^{\mathrm{T}}]^{\mathrm{T}}$.
- Network: graph+configuration

◇ Bearing vector:

$$g_{ij} = \frac{p_j - p_i}{\|p_j - p_i\|} \quad \forall (i,j) \in \mathcal{E}.$$

◇ An orthogonal projection matrix:

$$P_{g_{ij}} = I_d - g_{ij} g_{ij}^T,$$

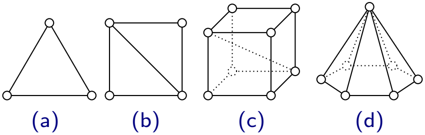


- $P_{g_{ij}}$ is symmetric positive semi-definite and $P_{g_{ij}}^2 = P_{g_{ij}}$
- $\mathrm{Null}(P_{g_{ij}}) = \mathrm{span}\{g_{ij}\} \Longleftrightarrow P_{g_{ij}} x = 0$ iff $x \parallel g_{ij}$ (important)

Bearing rigidity theory - Two problems

Two problems in the bearing rigidity theory

- How to determine the bearing rigidity of a given network?
- How to construct a bearing rigid network from scratch?



◇ Definition of bearing rigidity: shape can be uniquely determined by bearings
◇ Mathematical tool 1: bearing rigidity matrix
◇ Mathematical tool 2: bearing Laplacian matrix

◇ Mathematical tool 1: bearing rigidity matrix

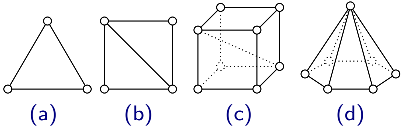
$$f(p) \triangleq \begin{bmatrix} g_1 \\ \vdots \\ g_m \end{bmatrix} \in \mathbb{R}^{dm}$$

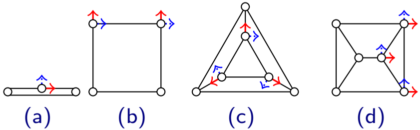
$$R(p) \triangleq \frac{\partial f(p)}{\partial p} \in \mathbb{R}^{dm \times dn}$$

$$df(p) = R(p)dp$$

Trivial motions: translation and scaling

Examples:





Condition for Bearing Rigidity

A network is bearing rigid if and only if $\mathrm{rank}(R) = dn - d - 1$.

Reference: S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-only formation stabilization,", *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1255-1268, 2016.

Bearing rigidity theory - Bearing Laplacian matrix

◇ Mathematical tool 2: bearing Laplacian matrix
◇ $\mathcal{B} \in \mathbb{R}^{dn \times dn}$ with the ijth subblock matrix as

$$[\mathcal{B}]_{ij} = \begin{cases} \mathbf{0}_{d\times d}, & i \neq j, (i,j) \notin \mathcal{E} \\ -P_{g_{ij}}, & i \neq j, (i,j) \in \mathcal{E} \\ \sum_{j \in \mathcal{N}_i} P_{g_{ij}}, & i \in \mathcal{V} \end{cases}$$

Condition for Bearing Rigidity

A network is bearing rigid if and only if $\mathrm{rank}(\mathcal{B}) = dn - d - 1$.

Reference: S. Zhao and D. Zelazo, "Localizability and distributed protocols for bearing-based network localization in arbitrary dimensions," Automatica, vol. 69, pp. 334-341, 2016.

Construction of bearing rigid networks

Definition (Laman Graphs)

A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is Laman if $|\mathcal{E}| = 2|\mathcal{V}| - 3$ and every subset of $k \geq 2$ vertices spans at most $2k - 3$ edges.

$\diamond$ Why consider Laman graphs: (i) favorable since edges distribute evenly in a Laman graph; (ii) widely used in, for example, distance rigidity; (iii) can be constructed by Henneberg Construction.

Definition (Henneberg Construction)

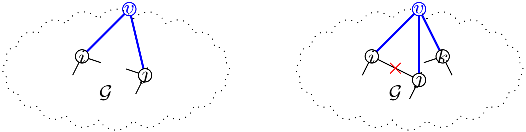
Given a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, a new graph $\mathcal{G}' = (\mathcal{V}', \mathcal{E}')$ is formed by adding a new vertex v to $\mathcal{G}$ and performing one of the following two operations:

(a) *Vertex addition*: connect vertex v to any two existing vertices $i, j \in \mathcal{V}$. In this case, $\mathcal{V}' = \mathcal{V} \cup \{v\}$ and $\mathcal{E}' = \mathcal{E} \cup \{(v, i), (v, j)\}$.

(b) *Edge splitting*: consider three vertices $i, j, k \in \mathcal{V}$ with $(i, j) \in \mathcal{E}$ and connect vertex v to i, j, k and delete (i, j). In this case, $\mathcal{V}' = \mathcal{V} \cup \{v\}$ and $\mathcal{E}' = \mathcal{E} \cup \{(v, i), (v, j), (v, k)\} \setminus \{(i, j)\}$.

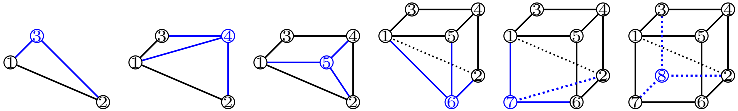
Bearing rigidity theory - Construction of networks

Two operations in Henneberg construction:



(a) Vertex addition (b) Edge splitting

Main Result: Laman graphs are generically bearing rigid in arbitrary dimensions.



Step 1: vertex addition Step 2: edge splitting Step 3: edge splitting Step 4: edge splitting Step 5: edge splitting Step 6: edge splitting

Reference: S. Zhao, Z. Sun, D. Zelazo, M. H. Trinh, and H.-S. Ahn, "Laman graphs are generically bearing rigid in arbitrary dimensions," in *Proceedings of the 56th IEEE Conference on Decision and Control, (Melbourne, Australia), December 2017. accepted*

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Bearing-only formation control - Control law

Nonlinear bearing-only formation control law

$$\dot{p}_i(t) = -\sum_{j\in\mathcal{N}_i} P_{g_{ij}(t)} g_{ij}^*, \quad i = 1, \ldots, n$$

- $p_i(t)$: position of agent i
- $P_{g_{ij}(t)} = I_d - g_{ij}(t)(g_{ij}(t))^{\mathrm{T}}$
- $g_{ij}(t)$: bearing between agents i and j at time t
- g_{ij}^*: desired bearing between agents i and j

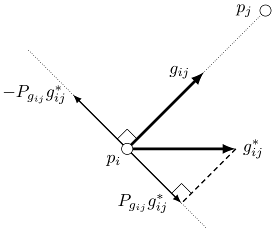


Figure: The geometric interpretation of the control law.

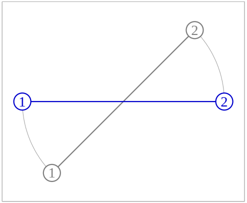


Figure: The simplest simulation example.

Show video

Bearing-only formation control - Stability analysis

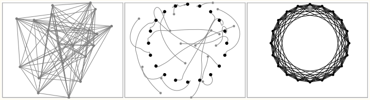
Centroid and Scale Invariance

- Centroid of the formation

$$\bar{p} \triangleq \frac{1}{n}\sum_{i=1}^{n} p_i$$

- Scale of the formation

$$s \triangleq \sqrt{\frac{1}{n}\sum_{i=1}^{n} \|p_i - \bar{p}\|^2}.$$



(a) Initial formation (b) trajectory (c) Final

Almost global convergence

- Two isolated equilibriums: one stable, one unstable

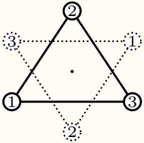


Figure: The solid one is the target formation.

Reference: S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-only formation stabilization," IEEE Transactions on Automatic Control, vol. 61, no. 5, pp. 1255-1268, 2016.

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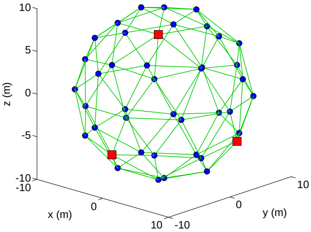
Bearing-based network localization

Distributed network localization

Given the inter-node bearings and some anchors, how to localize the network?

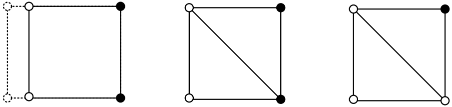


Two key problems

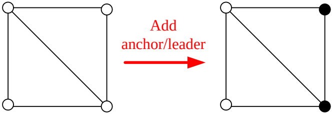
- Localizability: whether a network can be possibly localized?
- Localization algorithm: if a network can be localized, how to localize it?

Bearing-based network localization - Localizability

Not all networks are localizable:



From bearing rigidity to network localizability:

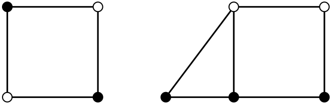


(a) infinitesimally bearing rigid

(b) localizable

Observations:

- bearing rigidity + two anchors $\Longrightarrow$ localizability
- bearing rigidity is sufficient but not necessary for localizability



(a) (b)

Bearing Laplacian: $\mathcal{B} \in \mathbb{R}^{dn \times dn}$ and the ijth subblock matrix of $\mathcal{B}$ is

$$[\mathcal{B}]_{ij} = \begin{cases} \sum_{j \in \mathcal{N}_i} P_{g_{ij}}, & i \in \mathcal{V}. \\ -P_{g_{ij}}, & i \neq j, (i,j) \in \mathcal{E}, \\ \mathbf{0}_{d \times d}, & i \neq j, (i,j) \notin \mathcal{E}, \end{cases}$$

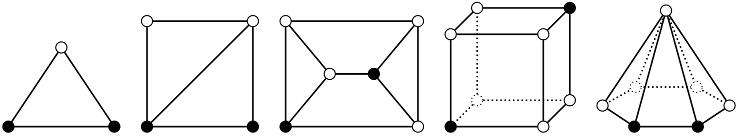
Bearing Laplacian is a *matrix-weighted Laplacian matrix*. The bearing Laplacian $\mathcal{B}$ can be partitioned into

$$\mathcal{B} = \begin{bmatrix} \mathcal{B}_{aa} & \mathcal{B}_{af} \\ \mathcal{B}_{fa} & \mathcal{B}_{ff} \end{bmatrix}$$

Necessary and sufficient condition

A network is localizable if and only if $\mathcal{B}_{ff}$ is nonsingular

Examples:



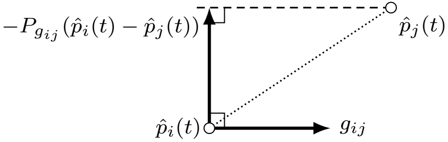
(a) (b) (c) (d) (e)

- ◇ If a network is localizable, then how to localize it?
- ◇ Localization protocol:

$$\dot{\hat{p}}_i(t) = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}}(\hat{p}_i(t) - \hat{p}_j(t)), \quad i \in \mathcal{V}_f.$$

where $P_{g_{ij}} = I_d - g_{ij} g_{ij}^T$.

- ◇ Geometric meaning:



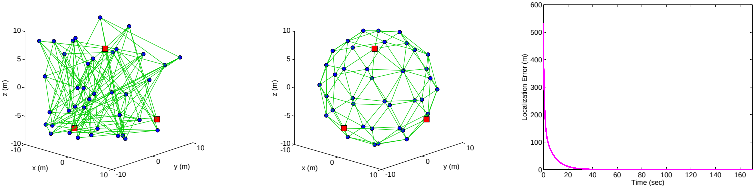
- ◇ Matrix form: $\dot{\hat{p}} = -\mathcal{B}\hat{p}$

Convergence

The protocol can globally localize a network if and only if the network is localizable.

Bearing-based network localization - Localization algorithm

Simulation:



(a) Random initial estimate

(b) Final estimate

(c) Total estimate error $\sum_{i\in\mathcal{V}_f}\|\hat{p}_i(t)-p_i\|$

S. Zhao and D. Zelazo, "Localizability and distributed protocols for bearing-based network localization in arbitrary dimensions," Automatica, vol. 69, pp. 334-341, 2016.

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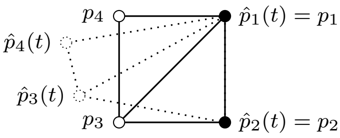
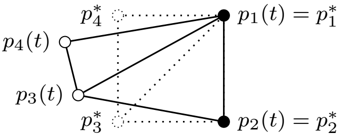
From network localization to formation control



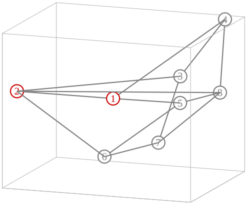
(a) Network localization

$\Rightarrow$

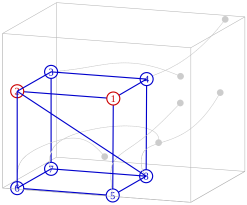


(b) Formation control

$$\dot{\hat{p}}_i(t) = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}} (\hat{p}_i(t) - \hat{p}_j(t)) \quad \Longrightarrow \quad \dot{p}_i(t) \quad = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}^*} (p_i(t) - p_j(t))$$



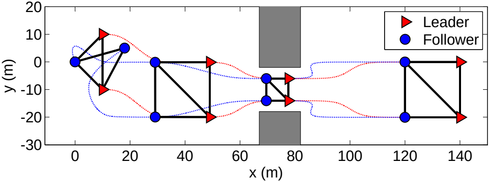
(a) Initial formation



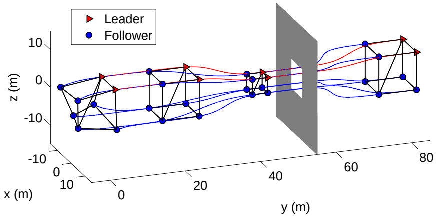
(b) Final formation

Bearing-based formation maneuver control

$$\dot{p}_i(t) = -\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\left[k_p(p_i(t)-p_j(t)) + k_I\int_0^t (p_i(\tau)-p_j(\tau))\mathrm{d}\tau\right]$$



$$\dot{p}_i(t) = v_i(t), \quad \dot{v}_i(t) = -\sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\left[k_p(p_i(t)-p_j(t)) + k_v(v_i(t)-v_j(t))\right]$$



S. Zhao and D. Zelazo, "Translational and scaling formation maneuver control via a bearing-based approach," *IEEE Transactions on Control of Network Systems*, , vol. 4, no. 3, pp. 429-438, 2017 S. Zhao and D. Zelazo, "Bearing Rigidity Theory and its Applications for Control and Estimation of Network Systems: Life beyond distance

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Formation control with motion constraints

How to handle motion constraints:

- nonholonomic constraint
- linear and angular velocity saturation
- obstacle avoidance and inter-neighbor collision avoidance

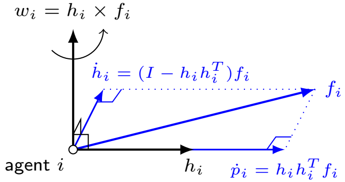
◇ The original gradient control law:

$$\dot{p}_i = f_i, \quad i \in \mathcal{V}$$

◇ The proposed modified gradient control law:

$$\begin{aligned} \dot{p}_i &= h_i h_i^T f_i, \\ \dot{h}_i &= (I - h_i h_i^T) f_i, \quad i \in \mathcal{V}. \end{aligned}$$

◇ Geometric interpretation:



Formation control with motion constraints

◇ The modified gradient control law:

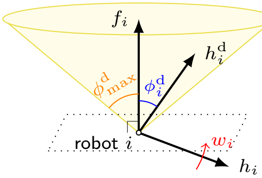
$$\dot{p}_i = h_i h_i^T f_i,$$
$$\dot{h}_i = (I - h_i h_i^T) f_i, \quad i \in \mathcal{V}.$$

◇ **A generalized version:**

$$\dot{p}_i = \kappa_i h_i h_i^T f_i,$$
$$\dot{h}_i = (I - h_i h_i^T) h_i^{\mathrm{d}},$$

where $\kappa_i(t) > 0$ and $h_i^{\mathrm{d}}(t) \in \mathbb{R}^d$ are time-varying.

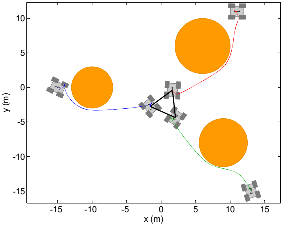
◇ Geometric interpretation:



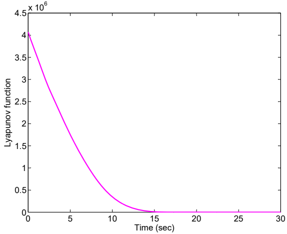
Reference: S. Zhao, D. V. Dimarogonas, Z. Sun, and D. Bauso, "A general approach to coordination control of mobile agents with motion constraints," *IEEE Transactions on Automatic Control*, accepted

Formation control with motion constraints - Simulation result

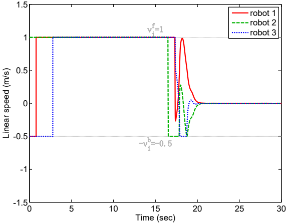
Distance-based formation control: unicycle robots, velocity saturation, obstacle avoidance



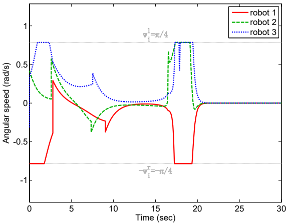
(a) Trajectory



(b) The Lyapunov function converges to zero.



(c) The linear velocity saturation



(d) The angular velocity

Formation control with motion constraints - on going research

- ◇ Integrate velocity obstacle with formation control
- ◇ Show video

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2 Bearing-only formation control

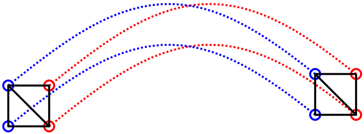
3 Bearing-based network localization

4 Bearing-based formation control

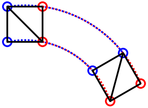
5 Formation control with motion constraints

6 Affine formation maneuver control

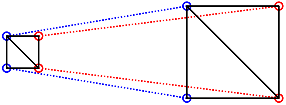
Different approaches lead to different maneuverability of the formation!



(a) Translational maneuver



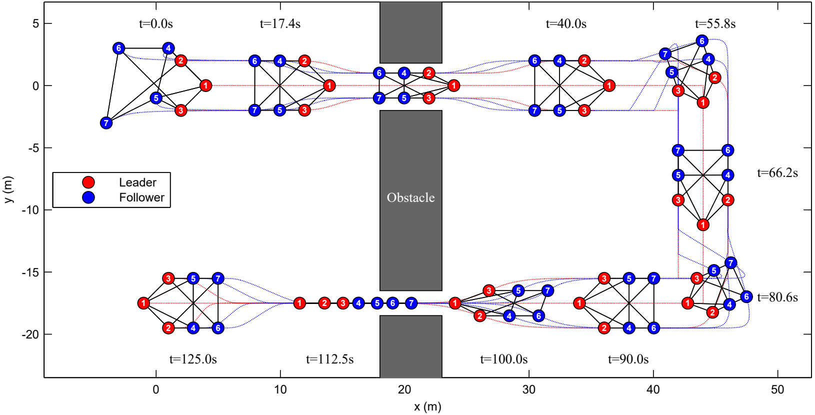
(b) Translational and rotational maneuver



(c) Translational and scaling maneuver

Can we achieve all of them simultaneously?

Affine formation maneuver control



Affine formation maneuver control

◇ Formation control law:

$$\dot{p}_i = -\sum_{j \in \mathcal{N}_i} \omega_{ij}(p_i - p_j), \quad i \in \mathcal{V}.$$

◇ The matrix-vector form is

$$\dot{p} = -(\Omega \otimes I_d)p.$$

◇ Key properties of Ω under the assumption:

- Stability of Ω: positive semi-definite
- Null space of Ω: $\mathrm{Null}(\Omega \otimes I_d) = \mathcal{A}(r)$

$$\begin{aligned}\mathcal{A}(r) &= \left\{p \in \mathbb{R}^{dn} : p_i = Ar_i + b, i \in \mathcal{V}, \forall A \in \mathbb{R}^{d\times d}, \forall b \in \mathbb{R}^d\right\} \\ &= \left\{p \in \mathbb{R}^{dn} : p = (I_n \otimes A)r + \mathbf{1}_n \otimes b, \forall A \in \mathbb{R}^{d\times d}, \forall b \in \mathbb{R}^d\right\}\end{aligned}$$

Reference: S. Zhao, "Affine formation maneuver control of multi-agent systems", *IEEE Transactions on Automatic Control*, conditionally accepted

show video

The end

Topics covered by this talk:

- Bearing rigidity theory
- Bearing-only formation control law
- Bearing-based network localization
- Bearing-based formation maneuver control
- Formation control with motion constraints
- Affine formation maneuver control

Thank you!

Related publication

- S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-only formation stabilization,", *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1255-1268, 2016.
- S. Zhao and D. Zelazo, "Localizability and distributed protocols for bearing-based network localization in arbitrary dimensions," *Automatica*, vol. 69, pp. 334-341, 2016.
- S. Zhao and D. Zelazo, "Translational and scaling formation maneuver control via a bearing-based approach," *IEEE Transactions on Control of Network Systems*, , vol. 4, no. 3, pp. 429-438, 2017.
- S. Zhao, D. V. Dimarogonas, Z. Sun, and D. Bauso, "A general approach to coordination control of mobile agents with motion constraints," *IEEE Transactions on Automatic Control*, accepted
- S. Zhao, "Affine formation maneuver control of multi-agent systems", *IEEE Transactions on Automatic Control*, conditionally accepted
- S. Zhao and D. Zelazo, "Bearing Rigidity Theory and its Applications for Control and Estimation of Network Systems: Life beyond distance rigidity", *IEEE Control Systems Magazine*, accepted