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Brief paper

Affine formation maneuver control of high-order multi-agent systems over directed networks☆

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ABSTRACT

To drive a group of agents to maneuver continuously with the desired collective forms, this paper addresses a distributed formation maneuver control problem of directed networked high-order multi-agent systems in arbitrary dimensions. Unlike the conventional methods where the target formation is time invariant, we propose an affine formation method based on the properties of affine transformation, in which the target formation can be time-varying and affinely transformed from the given nominal formation. This paper provides and proves a sufficient and necessary condition of achieving the directed graphical affine localizability, and it only needs that the leaders have a generic configuration and the followers are accessible to the subset of leaders. To achieve the whole formation maneuvers, assume that the leaders decide the whole formation's maneuver actions, then the control algorithms of the arbitrary-order integrator followers are proposed to track the time-varying target formation and the global convergence of tracking errors is also proved. Furthermore, this paper studies the practical problems of formation maneuvers when existing non-uniform time delays. Finally, both two-dimensional and three-dimensional simulation examples are given to demonstrate the effectiveness of theoretical results.

1. Introduction

This paper studies a multi-agent formation maneuver control task to distributedly steer the agents to maneuver as a whole such that the translation, orientation, scale, and other motions in arbitrary dimensions, and the desired geometric patterns with any given initial configurations can be achieved as well. The desired patterns and maneuvers are crucial for some complex tasks such as avoiding obstacles and passing narrow corridors.

Although there are numerous distributed formation control approaches in the existing literature, most of them cannot solve the formation maneuver control task studied in this paper. These methods can be classified by the definition of the desired geometric pattern of the target formation. Among three popular approaches in the recent surveys (Oh, Park, & Ahn, 2015; Zhu, Xie, Han, Meng, & Teo, 2017), e.g., the target formation is defined according to relative displacements (Dong & Hu, 2017), relative distances (Sun, Park, Anderson, & Ahn, 2017) or relative bearings (Zhao & Zelazo, 2019) of neighboring agents. The cases in the presence of time-varying target formation shapes have not yet been sufficiently tackled. Inspired by the shortcomings of the three schemes, scientists and engineers have tried to integrate the properties of special consensus into the design process of formation maneuver controllers. Then, many novel types of constant constraints are used to define the target formations like complex Laplacians (Lin, Wang, Han, & Fu, 2014) and stress matrices (Zhao, 2018). The control approach based on complex Laplacians can simultaneously perform translational, rotational, and scaling formation maneuvers (Han, Wang, Lin, & Zheng, 2016), but it is confined in the plane. The latter method is invariant to any affine transformation of required target formations in arbitrary dimensions. By combining the properties of both affine transformations and stress matrices, Zhao (2018) proposes an affine formation maneuver controller to achieve various maneuvers, such as a translation, rotation, scaling, or even shape deformation of the target formations.

Directed networks and high-order dynamical models for formation maneuvers are two challenging problems for real-world implementations, and they have not yet well solved. The stress

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matrices-based formation maneuver control protocols in Zhao (2018) are merely able to be applied to simple conditions, i.e., undirected graphs and low-order integrator dynamical models. Also, the networks between neighboring agents usually have orientations, and bidirectional measurements in Zhao (2018) are difficult to realize. For directed graphical cases, Xu, Zhao, Luo, and You (2018) proposes a signed Laplacian-based formation maneuver control scheme. The signed Laplacian offers a strong solver to obtain sorts of formation maneuvering behaviors. After investigating the existing literature for tackling multi-agent systems (Li, Ren, Liu, & Fu, 2013; Meng, Lin, & Ren, 2012; Ren, 2007, 2008; Song, Cao, & Yu, 2010), the high-order integral dynamic model therein has a means of being categorized into a particular case of general linear dynamics, which plays an essential role in the trajectory tracking problems as described in Cheng, Wang, Hou and Tan (2016) and Cheng, Wang, Ren and Hou (2016). In practice, the motion of every leader can be simplified to be a sequence of suitable reference points in corresponding dimensions, and the apt order trajectory that passes these points is generated by the spline or polynomial interpolation (Richter, Bry, & Roy, 2016; Xu, Lai, Li, Luo, & You, 2019). Then, the followers are controlled to track these generated trajectories of leaders. As a result, the formation maneuver control problems can be formulated regardless of whether the leaders and followers are homogeneous or heterogeneous. It should be mentioned that the leaders usually take up a small proportion of the whole formation, and for the maneuverability of formation, their trajectories can be generated by intelligent path planners.

We also tackle another two untouched practical problems of formation maneuvers in the existing work. Firstly, based on the distributed property along with exchanging networks, the performance of multi-agent formation controllers is usually subject to various time delays inside information flows. Existing time delay algorithms can be categorized as uniform and non-uniform types (Dong, Han, Li, Chen, & Ren, 2016; Dong, Li, Ren, & Zhong, 2015; Hou, Fu, Zhang, & Wu, 2017; Huang, Fang, Dou, & Chen, 2014). For the motivation of applications, time delays induced by measurement constraints are best to be modeled by non-uniform ones. Secondly, with proportional (P)-type or proportional–derivative (PD)-type protocols, the steady-state formation error induced by the leaders cannot be eliminated under some circumstances, e.g., occurring the constant inputs of leaders. To address this limitation, an integral term of the formation error is augmented, and the controllers become proportional–integral (PI)-type or proportional–integral–derivative (PID)-type. Motivated by the properties of PID-type algorithms in Lombana and Di Bernardo (2015, 2016), if leaders and followers are heterogeneous and the trajectories of leaders have the higher order, then high-order integral and derivative terms of the formation error can be integrated into the controllers.

The main contributions include three aspects:

(i) The necessary and sufficient condition for the directed graphical affine localizability is analyzed and proved, which holds in arbitrary dimensions. Superior to our previous work (Xu et al., 2018; Zhao, 2018), a more flexible condition of only the generic partial nominal configuration of the leader subsystem is provided.

(ii) The motions of leaders can determine the whole formation maneuver actions, which are represented as connected polynomial trajectories. We then propose a PI^n-type linear formation maneuver control law for the high-order integral followers. The global convergence analysis of tracking errors is also proved.

(iii) We deal with the affine formation maneuvers in the presence of practical conditions, e.g., time delays exist during the information exchange. Unlike time-domain Lyapunov-type functional approaches (Dong et al., 2016, 2015), the upper bound of delays is proved in the frequency domain, and the delays can be non-uniform and time-varying.

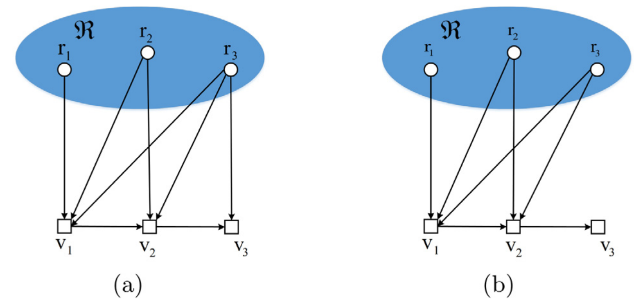


Fig. 1. Two examples to illustrate 3-reachable and non 3-reachable conditions. (a) All the nodes v_1, v_2, and v_3 are 3-reachable form the root set $\mathcal{R} = \{r_1, r_2, r_3\}$, and there exists a spanning 3-tree. (b) Node v_3 is not 3-reachable form set $\mathcal{R}$ after removing node v_2.

Notations: Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{C}$ denote the set of complex numbers, $\mathbf{1}_n$ stand for a n-dimensional vector of ones, and I_n represent a $n \times n$ identity matrix. Denote $\otimes$ as the Kronecker product, diag$(\cdot)$ the diagonal matrix, Re$(\cdot)$ the real part of a complex number.

2. Preliminaries and problem statement

2.1. Directed graph theory

A *directed graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ contains a node set $\mathcal{V} = \{1, 2, \ldots, N\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Denote $\mathcal{N}_i$ as the in-neighbor set of the node i, where $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$. A *path* is a sequence of edges in a directed graph of the form $(i, i+1), (i+1, i+2), \ldots$. Throughout this paper, it is assumed that the directed graph is a fixed topology and does not have any self-loops.

For a directed graph $\mathcal{G}$, we cite the definitions of κ*-reachable*, κ*-rooted*, and *spanning* κ*-tree* from Lin, Wang, Chen, Fu, and Han (2016). The corresponding examples of 3-reachable and non 3-reachable graphs are shown in Fig. 1.

In the sequel, we introduce the notion of a special Laplacian: a *signed Laplacian* L^s is associated to a graph with real weights $\omega_{ij} \neq 0$ that may be positive or negative. For the Laplacian L^s, the off-diagonal entry $L^s(i, j) = -\omega_{ij}$ if $j \in \mathcal{N}_i$ and 0 otherwise, whereas the diagonal element $L^s(i, i) = \sum_{k \in \mathcal{N}_i} \omega_{ik}$. Besides, L^s is normally an asymmetric matrix and $L^s \mathbf{1}_N = 0$.

2.2. Problem statement

In this paper, we consider a group of N mobile agents in $\mathbb{R}^d$ and $d \geq 2$. Define the leader subset containing N_ℓ agents as $\mathcal{V}_\ell = \{1, 2, \ldots, N_\ell\}$, and the follower subset containing $N_f = N - N_\ell$ agents as $\mathcal{V}_f = \mathcal{V} \backslash \mathcal{V}_\ell$. The whole group of positions $y = [y_1^T, y_2^T, \ldots, y_N^T] \in \mathbb{R}^{dN}$ of N agents can constitute a configuration. The positions of leaders are $y_\ell = [y_1^T, y_2^T, \ldots, y_{N_\ell}^T]^T$ and those of followers are $y_f = [y_{N_\ell+1}^T, y_{N_\ell+2}^T, \ldots, y_N^T]^T$ respectively, thus $y = [y_\ell^T, y_f^T]^T$.

The agents exchange information via a directed interaction graph $\mathcal{G}$, and suppose that every leader does not interact with the other leaders and access the information from the followers. A formation $(\mathcal{G}, y)$ represents a directed graph $\mathcal{G}$ with the position y_i mapped from the ith node. Then the *nominal formation* associated with $\mathcal{G}$ is defined as $(\mathcal{G}, r)$, and $r = [r_1^T, r_2^T, \ldots, r_N^T]^T = [r_\ell^T, r_f^T]^T \in \mathbb{R}^{dN}$ is constant and called as *nominal configuration*. The *affine* *image* of the nominal configuration r is defined as $\mathcal{A}(r) = \{y \in \mathbb{R}^{dN} : y = (I_N \otimes A)r + \mathbf{1}_N \otimes b, \forall A \in \mathbb{R}^{d\times d}, \forall b \in \mathbb{R}^d\}$, where (A, b) is the *affine transformation*.

In the sequel, we introduce the *affine span* as $\mathcal{S} = \{\sum_{i=1}^N a_i y_i : a_i \in \mathbb{R}$ for all i and $\sum_{i=1}^N a_i = 1\}$. The dimension of the linear space is just the dimension of the affine span. If the dimension of the affine span is d, then we say that these positions affinely span in $\mathbb{R}^d$. Additionally, if these scalars $\{a_i\}_{i=1}^N$ except all zeros cannot satisfy $\sum_{i=i}^N a_i y_i = 0$ and $\sum_{i=i}^N a_i = 0$, then we call this condition as *affinely independent*. With the definition of a *configuration matrix* in Zhao (2018), it implies that there exist at most $d+1$ positions and then $\{y_i\}_{i=1}^N$ are affinely independent in $\mathbb{R}^d$.

Let differentiable $y_i(t) \in \mathbb{R}^d$ as the position of the ith agent at time t. Then, assume that the leader $i \in \mathcal{V}_\ell$ can move along the prescribed nth-order polynomial trajectory continuously

$$y_i(t) = \mathrm{a}_0^i + \mathrm{a}_1^i t + \cdots + \mathrm{a}_n^i t^n, \tag{1}$$

where $\mathrm{a}_k^i \in \mathbb{R}^d$ with $k = 0, 1, \ldots, n$. Then, the *time-varying target formation* is $y^*(t) = [I_d \otimes A(t)]\, r + \mathbf{1}_N \otimes b(t)$, where $A(t) \in \mathbb{R}^{d\times d}$ and $b(t) \in \mathbb{R}^d$ are continuous for t and time-varying, $y^*(t)$ is in $A(r)$ for all t.

In this paper, we want to solve the following problems:

Problem 1. Over the directed interaction graphs, the formation maneuvers for high-order multi-agent systems can be achieved such that all agents are always inside the target formation.

Problem 2. The corresponding affine formation maneuver controller under the non-uniform time delay graphical condition also needs to be designed.

3. Main results

3.1. Properties of affine localizability

Localizability identifies whether or not a network can be possibly localized by the leader locations and inter-neighbor relative information. Based on network localizability and affine transformation, the nominal formation $(\mathcal{G}, r)$ is said to be *affinely localizable* if the following two conditions are satisfied simultaneously: (i) for any $y = [y_\ell^T, y_f^T]^T \in \mathcal{A}(r)$ in $\mathbb{R}^{dN}$, y_f can be determined by y_ℓ uniquely; (ii) for $\mathcal{G}$ and y, there is a signed Laplacian $L^s \in \mathbb{R}^{N\times N}$ associated with $\mathcal{G}$ such that $(L^s \otimes I_d)y = 0$. In this definition, the first condition is for selecting leaders, and the second one considers the directed graphical condition. In the sequel, we make an assumption as follows, and a theorem of the necessary and sufficient condition to fulfill the affine localizability also can be deduced.

Assumption 1. The given nominal configuration r_ℓ of N_ℓ leaders satisfies $\{r_i\}_{i\in\mathcal{V}_\ell}$ affinely span in $\mathbb{R}^d$.

Theorem 1. *Under Assumption 1, the given nominal formation $(\mathcal{G}, r)$ of N agents in $\mathbb{R}^d$ is affinely localizable if and only if $\mathcal{V}_\ell$ in $\mathcal{G}$ has $d+1$ leaders and every follower in $\mathcal{V}_f$ is $(d+1)$-reachable from $\mathcal{V}_\ell$.*

Proof. (Sufficiency) The underlying directed graph $\mathcal{G}$ must be $(d+1)$-rooted, and it has a spanning $(d+1)$-tree under Lemma 2.1 in Lin et al. (2016). There exist $N_\ell(= d+1)$ leaders inside $\mathcal{V}_\ell$ and $N_f(= N-d-1)$ followers inside $\mathcal{V}_f$. Suppose that there is a signed Laplacian L^s associated with $\mathcal{G}$, and it must contain $d+1$ all zero rows corresponding to $\mathcal{V}_\ell$. Then, L_s can be partitioned into the following four blocks

$$L^s = \left[\begin{array}{c|c} 0_{\ell\ell}^{(d+1)\times(d+1)} & 0_{\ell f}^{(d+1)\times(N-d-1)} \\ \hline L_{f\ell}^{s\,(N-d-1)\times(d+1)} & L_{ff}^{s\,(N-d-1)\times(N-d-1)} \end{array}\right].$$

With Lemma 4.1 in Lin et al. (2016), it implies that $\text{rank}(L^s) = N-d-1$ and L_{ff}^s is nonsingular. By introducing $d+1$ linearly independent bases of L^s, we can deduce that the rank of $L_{f\ell}^s$ is $d+1$. Since $\mathcal{G}$ has a spanning $(d+1)$-tree, there are exactly $d+1$ in-neighbors for these nonroot nodes of followers. For the given nominal configuration r, assume that there exists another signed Laplacian $L^{s'}$ with $\mathcal{G}'$ satisfying $(L^{s'} \otimes I_d)r = 0$. Under Assumption 1, $\{r_i\}_{i=1}^N$ as well as $\{r_i\}_{i\in\mathcal{V}_\ell}$ affinely span in $\mathbb{R}^d$, and $L^{s'}$ has the same zero or nonzero pattern of elements as L^s. It can be obtained that $\text{rank}(L^{s'}) = \text{rank}(L^s) = N-d-1$ satisfies. Besides, $\mathcal{G}'$ is a subgraph inside $\mathcal{G}$, $\mathcal{G}'$ has weights of additional edges in $\mathcal{G}$ being zero. By using the fact that either a polynomial is zero or it is not zero almost everywhere, we can obtain that $(L^s \otimes I_d)r = 0$.

In the sequel, under Assumption 1 and Lemma 3 in Zhao (2018), $\mathcal{A}(r)$ has the same dimension of the null space of $L^s \otimes I_d$. For any $y \in \mathcal{A}(r)$, we can obtain that $(L_{f\ell}^s \otimes I_d)y_\ell + (L_{ff}^s \otimes I_d)y_f = 0$. Thus, y_f can be uniquely determined by y_ℓ if and only if L_{ff}^s is nonsingular. It implies that y_f can be calculated by $y_f = [-(L_{ff}^{s^{-1}} L_{f\ell}^s) \otimes I_d]y_\ell$ uniquely. Above all, the affine localizability of the given nominal formation $(\mathcal{G}, r)$ of N agents can be realized.

(Necessity) Here, we use the positions $y, r \in \mathbb{R}^{dN}$ as the specific case to make the corresponding contradictions. If the given nominal formation $(\mathcal{G}, r)$ is affinely localizable, then there exists a signed Laplacian L^s associated to $\mathcal{G}$ satisfying $(L^s \otimes I_d)y = 0$ for $y \in \mathcal{A}(r)$. Under Assumption 1, for any $y \in \mathcal{A}(r)$, there is (A, b) satisfying $y_i = Ar_i + b$, $i \in \mathcal{V}_\ell$ and $y_i = Ar_i + b$, $i \in \mathcal{V}_f$. Suppose that $\{r_i\}_{i\in\mathcal{V}_\ell}$ cannot affinely span in $\mathbb{R}^d$, there must exist infinite solutions of the affine transformation. This implication contradicts Assumption 1, and there exists another value of p_f, thus the affine localizability cannot be achieved and the contradiction occurs.

With Lemma 1 in Zhao (2018), it implies that $\text{rank}(\bar{C}(r_\ell)) = d+1$ and the minimum number of leaders is $d+1$. Moreover, if there are more than $d+1$ leaders in $\mathbb{R}^d$, an overdetermined linear system always occurs, and these positions of leaders need to depend on each other. If there exists a node in the set $\mathcal{R}$ and it is not a root. Then, under this condition that the leader number must be less than $d+1$, which contradicts the inference that the minimum number of leaders is $d+1$. Therefore, $\mathcal{R}$ must be the root set, and there are $d+1$ leaders exactly. Besides, by removing any path, it can be verified that there does not exist any follower that is not $(d+1)$-reachable from $\mathcal{V}_\ell$. $([L_{f\ell}^s, L_{ff}^s, 0] \otimes I_d)y = 0$, and the zero vector 0 has corresponding rows to these non $(d+1)$-reachable followers. Then, we can find that $[L_{f\ell}^s, L_{ff}^s, 0]$ is not full rank. There are $d+1$ leaders, and $\text{rank}(L^s) \le N-d-2$. Since the affine localizability can be achieved, $\text{rank}(L^s) = N-d-1$. The contradiction occurs, thus every follower must be $(d+1)$-reachable from the leader subset $\mathcal{V}_\ell$. Here we complete the proof. □

Now, another assumption is provided about the directed graphical condition of the nominal formation.

Assumption 2. Assume that the root set of given graph $\mathcal{G}$ of N agents contains $d+1$ leaders, and each follower can be $(d+1)$-reachable from the leaders.

Assumptions 1 and 2 imply a critical mathematical premise: The block of the signed Laplacian L_{ff}^s needs to be nonsingular. Besides, the number of agents $N \ge d+2$ since there exists at least one follower. Given a nominal formation $(\mathcal{G}, r)$ that satisfies

Assumptions 1–2, it has the property of $(L^s \otimes I_d)r = 0$, then these off-diagonal weights ω_{ij} can be computed by $\sum_{j\in\mathcal{N}_i} \omega_{ij}(r_j - r_i) = 0, i \in \mathcal{V}_f$, and the solution of L^s may be not unique. In this paper, we assume that the formation of leaders is always inside the desired target formation, i.e., $y_\ell(t) = y_\ell^*(t)$ for all t. Then, the control objective of formation maneuvers is modified to design the control law $u_i(t)$ for these followers to achieve $y_f(t) = y_f^*(t)$ as $t \to \infty$.

3.2. Affine formation maneuver control law

For a real-world task, we cannot exactly obtain the exact dynamical models of leaders and only take the measured trajectories of leaders (1) as the reference signals. The mth-order translational dynamics of the ith follower can be described as

$$\mathfrak{D}^m y_i(t) = u_i(t),\ i \in \mathcal{V}_f \tag{2}$$

where $y_i(t) \in \mathbb{R}^d$ represents the position vector, $m \in \mathbb{Z}^+$ denotes the relative derivative degree, and $u_i(t) \in \mathbb{R}^d$ is the corresponding control input. Here, we set the symbol $\mathfrak{D}$ as the differential operator, i.e., $\mathfrak{D}y_i(t) = \dot{y}_i(t)$ and $\mathfrak{D}^n y_i(t) = \mathfrak{D}(\mathfrak{D}^{n-1}y_i(t)) = y_i^n(t)$. The inversion of $\mathfrak{D}$ can be an integral operation like $\mathfrak{D}^{-1}y_i(t) = \int_0^t y_i(s)\mathrm{d}s$.

We propose a PI^n-type affine formation maneuver control law for the followers $i \in \mathcal{V}_f$ as

$$u_i(t) = -\sum_{l=0}^{l_m-1} k_l \mathfrak{D}^{m-l-1} d_i \sum_{j\in\mathcal{N}_i} \omega_{ij} \left(y_i(t) - y_j(t)\right), \tag{3}$$

where $l_m = \max\{m, n+1\}$, and $\{k_l | l = 0, 1, \ldots, l_m - 1\}$, d_i, ω_{ij} are parameters to be determined. For instance, if the relative derivative degree $m = 1$ and the order of leaders' trajectories $n \le 1$, then we can find $l_m = n + 1$. Before moving forward, we rewrite (1) and (2) to the matrix–vector form as

$$\begin{aligned} &\mathfrak{D}\vartheta_i(t) = (E \otimes I_d)\,\vartheta_i(t), && i \in \mathcal{V}_\ell, \\ &\mathfrak{D}\vartheta_i(t) = (E \otimes I_d)\,\vartheta_i(t) + (F \otimes I_d)\,\upsilon_i(t), && i \in \mathcal{V}_f, \end{aligned} \tag{4}$$

where $\vartheta_i(t) = [y_i^T(t), \mathfrak{D}y_i^T(t), \ldots, \mathfrak{D}^{l_m-1}y_i^T(t)]^T$, $\upsilon_i(t) = \mathfrak{D}^{l_m-1}u_i(t)$,

$$E = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{l_m \times l_m},\ F = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^{l_m}.$$

Under Assumptions 1–2 such that L_{ff}^s satisfies nonsingularity, there exists a permutation matrix M such that all the leading principal minors of $ML_{ff}^sM^T$ are nonzero. According to Theorem 1 in Ballantine (1970), it can be obtained that a diagonal matrix D' makes all the eigenvalues of $D'ML_{ff}^sM^T$ locate in the right-half plane. In addition, M has the property of $M^{-1} = M^T$, thus $D'ML_{ff}^sM^T = M(M^TD'ML_{ff}^s)M^T$. It implies that both $D'ML_{ff}^sM^T$ and $M^TD'ML_{ff}^s$ have the same eigenvalues. Since $M^TD'M$ is also diagonal, then let $D'' = M^TD'M$, and the diagonal entries of D corresponding to zero eigenvalues constitute I_{d+1}. The form of D can be partitioned into $D = \mathrm{diag}(I_{d+1}, D'')$. After premultiplying the diagonal D, these eigenvalues of DL^s have $d+1$ zeros and the rest with positive real parts. This diagonal matrix D is called as the stabilizing matrix, and by partitioning DL^s into blocks, then we define $\bar{L}_{f\ell}^s = D''L_{f\ell}^s$ and $\bar{L}_{ff}^s = D''L_{ff}^s$.

Here, we define $\Theta_\ell(t) = [\vartheta_1^T(t), \ldots, \vartheta_{N_\ell}^T(t)]^T$, $\Theta_f(t) = [\vartheta_{N_\ell+1}^T(t), \ldots, \vartheta_N^T(t)]^T$, and $K = [k_{l_m}, k_{l_m-1}, \ldots, k_0]$. Since the leaders satisfy $\Theta_\ell(t) = \Theta_\ell^*(t)$ always holds during the movement. For the followers $\Theta_f^*(t) = [-(\bar{L}_{ff}^{s^{-1}}\bar{L}_{f\ell}^s) \otimes I_{l_md}]\Theta_\ell^*(t)$, we define the tracking error of the followers as $\hat{\Theta}_f(t) = \Theta_f(t) - \Theta_f^*(t)$. In the sequel, we can obtain

$$\mathfrak{D}\hat{\Theta}_f(t) = \left(I_{N_f} \otimes E \otimes I_d - \bar{L}_{ff}^s \otimes FK \otimes I_d\right)\hat{\Theta}_f(t). \tag{5}$$

The transformation of $\bar{L}_{ff}^s$ needs to be given here. Denote a nonsingular matrix $U_f \in \mathbb{C}^{N_f\times N_f}$ satisfying $U_f^{-1}\bar{L}_{ff}^sU_f = J_f$, where J_f is the Jordan canonical form of the matrix $\bar{L}_{ff}^s$ with diagonal entries $\lambda_i, i \in \mathcal{V}_f$ satisfying $\mathrm{Re}(\lambda_1) \le \cdots \le \mathrm{Re}(\lambda_{N_f})$. Then, we define $\tilde{\Theta}_f(t) = (U_f^{-1} \otimes I_{l_md})\hat{\Theta}_f(t)$. By rewriting the equation of (5) as

$$\begin{aligned} \mathfrak{D}\tilde{\Theta}_f(t) &= \left(I_{N_f} \otimes E \otimes I_d - J_f \otimes FK \otimes I_d\right)\tilde{\Theta}_f(t) \\ &= \left[\left(I_{N_f} \otimes E - J_f \otimes FK\right) \otimes I_d\right]\tilde{\Theta}_f(t). \end{aligned} \tag{6}$$

Then a theorem is given as below, which proves the convergence of tracking error $\hat{\Theta}_f(t)$ under the proposed control law (3).

Theorem 2. *Under Assumptions 1–2, if the leaders move along the polynomial trajectories $y_\ell^*(t)$, then the tracking error $\hat{\Theta}_f(t)$ of the followers under the control law (3) converges globally and exponentially to zero if and only if $E - \lambda_iFK$ is Hurwitz, $\{\lambda_i, i \in \mathcal{V}_f\}$ are the diagonal entries of Jordan canonical form J_f of $\bar{L}_{ff}^s$.*

Proof. If and only if $E - \lambda_iFK$ for all the followers $i \in \mathcal{V}_f$ can achieve Hurwitz, then we can find that $I_{N_f} \otimes E - J_f \otimes FK$ is also Hurwitz. From (6), it implies that $\tilde{\Theta}_f(t)$ can converge globally and exponentially to zero. In the sequel, by utilizing the nonsingular matrix U_f to transfer (6) back to (5), we can obtain that $I_{N_f} \otimes E - \bar{L}_{ff}^s \otimes FK$ satisfies Hurwitz. Therefore, the actual tracking error $\hat{\Theta}_f(t)$ of the followers also can achieve exponential stability to origin. Here the proof completes. □

Since Theorem 2 has been proven, a method to design the control gain matrix K needs to be provided as the following theorem.

Theorem 3. *If $E - \lambda_iFK, i \in \mathcal{V}_f$ is Hurwitz, then $K = c[\mathrm{Re}(\lambda_1)]^{-1} R^{-1}F^TP$, where a threshold constant $c > 0.5$, and P is the positive solution of the algebraic Riccati equation $PE + E^TP - PFR^{-1}F^TP + Q = 0$, where $R = R^T > 0$, and $Q = Q^T > 0$.*

Proof. Consider the stability of the following subsystems of the followers $i \in \mathcal{V}_f$ as

$$\dot{\psi}_i(t) = (E - \lambda_iFK)\psi_i(t). \tag{7}$$

By establishing a Lyapunov function as

$$V_i(t) = \psi_i^H(t)P\psi_i(t). \tag{8}$$

Since $K = c[\mathrm{Re}(\lambda_1)]^{-1}R^{-1}F^TP$ and $PE + E^TP = PER^{-1}F^TP - Q$, one has

$$\begin{aligned} \dot{V}_i(t) =& \psi_i^H(t)\{(1 - 2c[\mathrm{Re}(\lambda_1)]^{-1}\mathrm{Re}(\lambda_i))\cdot \\ & (PFR^{-1}F^TP) - Q\}\psi_i(t). \end{aligned} \tag{9}$$

Under Assumptions 1–2 such that L_{ff}^s is nonsingular, and further $\bar{L}_{ff}^s$ has all positive real-part eigenvalues. Then, according to the definition of $\lambda_i, i \in \mathcal{V}_f$, it has the property $0 < \mathrm{Re}(\lambda_1) \le \mathrm{Re}(\lambda_i)$. If $\psi_i(t)$ is stable, then $\dot{V}_i(t) < 0$. Since $P > 0$ and $R = R^T > 0$, it implies that $PFR^{-1}F^TP \ge 0$. It is also known that $Q = Q^T > 0$, thus $1 - 2c[\mathrm{Re}(\lambda_1)]^{-1}\mathrm{Re}(\lambda_i) < 0$ should be satisfied. With $c > 0.5$, it can be obtained that $\dot{V}_i(t) < 0$ and $\psi_i(t)$ is stable. Therefore, the tracking error $\hat{\Theta}_f(t)$ of the followers under the control law (3) converges globally and exponentially to zero. □

Remark 1. A leader–follower structure is adopted in this paper, and the affine formation control is similar to the containment control in Cheng, Wang, Ren et al. (2016) since there are multiple leaders from the directed graphical structure. However, the containment control of the followers cannot be used to form a particular shape, and their positions are randomly localized inside the convex hull of the leaders. The affine formation control approach can be applied to required target formation shapes, whether or not the followers are inside the convex hull spanned by the leaders, and the followers can converge into the specified target formation. Besides, compared with the control law in Cheng, Wang, Ren et al. (2016), we need to premultiply an additional parameter d_i in order to construct a stabilizing diagonal matrix D. Then, D can place the negative real-part eigenvalues of L^s_{ff} into the right half plane.

Remark 2. In practical implementation of applying the *PI* strategy, the tracking errors of the followers may be amplified by the small measurement errors. This might render the integral terms unable to converge, and the integral action can be seen as the secondary frequency control (Emma & Sandberg, 2017). The role of the integral action is to eliminate any stationary frequency control errors induced by *P* or *PD* control, and the transient performance also improves simultaneously. In practice, anti-windup strategies (saturations) can be added into the integral terms or higher-order actions (e.g. PI^n) can be used. For the condition of installing relatively noisy measurement sensors, the *PI* strategy can be further exploited into a multi-layer network structure as (Lombana & Di Bernardo, 2016) by placing the proportional action and the integral action on different directed edges between neighboring agents respectively.

Remark 3. In this brief paper, we only consider that the heterogeneity may exist between the leader subsystem and the follower subsystem, thus these leaders are homogeneous in the leader subsystem and can only have different coefficients of the polynomial, i.e., moving along their respective trajectories.

Remark 4. In reality, these differential items may be hard to access, and the corresponding state estimators should be designed to compensate the control performance. For the leaders, they should be aware of own states like current positions and corresponding any-order derivatives precisely. Thus, the leaders do not need the estimators and can send their states to neighbors directly. For the ith follower, we define the estimated state $z_i(t) = [z^T_{i1}(t), z^T_{i2}(t), \cdots, z^T_{im}(t)]^T \in \mathbb{R}^{md}$. Besides, for the ith leader, it implies that $z_i(t) = [y^T_i(t), \mathfrak{D}y^T_i(t), \cdots, \mathfrak{D}^{m-1}y^T_i(t)]^T \in \mathbb{R}^{md}$. Then, the state estimator of the ith follower is proposed as

$$\mathfrak{D}z_i(t) = \left(\bar{E}\otimes I_d\right)z_i(t) + \left(\bar{F}\otimes I_d\right)u_i(t) - \left(\bar{K}\otimes I_d\right)\cdot d_i\sum_{j\in\mathcal{N}_i}\omega_{ij}\left[(G\otimes I_d)\left(z_i(t)-z_j(t)\right) - \left(y_i(t)-y_j(t)\right)\right], \tag{10}$$

where the gain matrix of the estimator is $\bar{K} \in \mathbb{R}^m$, $G = [1, 0, \ldots, 0] \in \mathbb{R}^{1\times m}$, and $\bar{E} \in \mathbb{R}^{m\times m}$ and $\bar{F} \in \mathbb{R}^m$ have the same arrangement of elements as E and F.

For each agent $i \in \mathcal{V}$, after introducing the estimator, let $z_i(t) = [z^T_{i1}, \ldots, z^T_{im}(t)]^T$, then we can replace $(\mathfrak{D}^l y_i(t) - \mathfrak{D}^l y_j(t))$ with $(z_{il}(t) - z_{jl}(t))$ for $l = 1, 2, \ldots, m-1$. Thus, for the ith follower, the tracking control law (3) can be modified as

$$u_i(t) = -d_i\sum_{j\in\mathcal{N}_i}\omega_{ij}\left(\sum_{l=m-1}^{l_m-1}k_l\left(y_i(t)-y_j(t)\right) + \sum_{l=0}^{m-2}k_l\left(z_{i(m-l)}(t)-z_{j(m-l)}(t)\right)\right), \tag{11}$$

where convergence of tracking error $\hat{\Theta}_f(t)$ of the followers can be proved by the input-to state stability in Khalil (2001).

3.3. Formation control subject to time delays

Suppose that there exist non-uniform time delays inside the network, the formation control problem would be more complicated. We propose the following time delay affine formation maneuver control law based on (3) for the ith follower as

$$u_i(t) = -\sum_{l=0}^{l_m-1}k_l\mathfrak{D}^{m-l-1}d_i\sum_{j\in\mathcal{N}_i}\omega_{ij}\left(y_i(t) - y_j(t-\tau_{ij}(t))\right), \tag{12}$$

where $l_m = \max\{m, n+1\}$, $\tau_{ij}(t)$ is the non-uniform transmitted information delay from the jth agent to the ith agent at time t.

Here, we partition $\bar{L}^s_{ff}$ into two parts as $\bar{L}^s_{ff} = D_f - A_f$, where D_f is the diagonal part of $\bar{L}^s_{ff}$, and A_f presents the adjacent matrix. In the sequel, denote $\tau_k(t) \in \{\tau_{ij}(t)\}$, $k \in \mathcal{Q}$, $\mathcal{Q} = \{1, 2, \ldots, q\}$, $q \le (N_\ell N_f + N_f(N_f-1)/2)$, and $A_k = [A_{kij}]$ is a matrix defined as A_{kij} if $i \ne j$ and $\tau_k(\cdot) = \tau_{ij}(\cdot)$, otherwise $A_{kij} = 0$. Besides, it has $\sum_{k=1}^{q}A_k = A_f$. If the network is a fully connected directed graph, then the total number q of different time delays attains its maximum, i.e., $N_\ell N_f + N_f(N_f-1)/2$. Another assumption about the time-varying delay is given as below.

Assumption 3. Assume that there exists the time-varying delay $\tau_{ij}(t)$ from the agent j to i at time t, $\tau_k(t) \in \{\tau_{ij}(t)\}$ for $k \in \mathcal{Q}$, $0 \le \tau_k(t) \le \tau_0$, $\dot{\tau}_k(t) \le c_k$, τ_0 is the maximum delay, and $\{c_k\}$ are constants.

According to (5) and (12), consider the stabilities of the following N_f time delay subsystems for $i \in \mathcal{V}_f$

$$\mathfrak{D}\hat{\Theta}^{(p)}_f(t) = \Gamma\hat{\Theta}^{(p)}_f(t) + \sum_{k=1}^{q}H_k\hat{\Theta}^{(p)}_f(t-\tau_k(t)), \tag{13}$$

where $\hat{\Theta}^{(p)}_f(t)$ is one dimensional component of $\hat{\Theta}_f(t)$ with $p \in \{1, 2, \ldots, d\}$, $\Gamma \in \mathbb{R}^{l_mN_f\times l_mN_f}$ and $H_k \in \mathbb{R}^{l_mN_f\times l_mN_f}$,

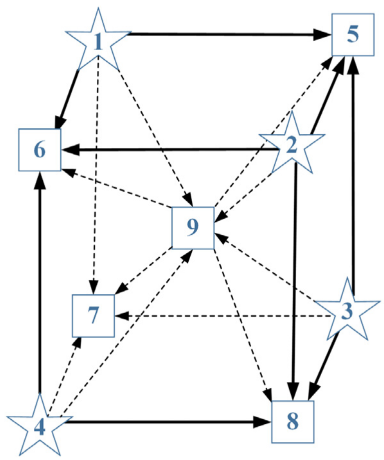
$$\Gamma = \begin{bmatrix} 0_{N_f} & I_{N_f} & \cdots & 0_{N_f} \\ 0_{N_f} & 0_{N_f} & \cdots & 0_{N_f} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{N_f} & 0_{N_f} & \cdots & I_{N_f} \\ -k_0D_f & -k_1D_f & \cdots & -k_{l_m-1}D_f \end{bmatrix},$$

$$H_k = \begin{bmatrix} 0_{N_f} & 0_{N_f} & \cdots & 0_{N_f} \\ 0_{N_f} & 0_{N_f} & \cdots & 0_{N_f} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{N_f} & 0_{N_f} & \cdots & 0_{N_f} \\ k_0A_k & k_1A_k & \cdots & k_{l_m-1}A_k \end{bmatrix}.$$

In the following, we tackle that problem with time-varying delays based on the small-μ stability theorem (Gu, Chen, & Kharitonov, 2003). To apply this theorem, we should transfer the delay system to a linear time-invariant plant and an upper bounded time delay operator. By the Laplace transformation of (13), we can obtain the following feedback interconnection of a linear time-invariant plant along with a delay operator as

$$\tilde{\Theta}^{(p)}_f = \mathfrak{G}(s)\Delta(\tilde{\Theta}^{(p)}_f), \tag{14}$$

where $\tilde{\Theta}^{(p)}_f = [\tilde{\Theta}^{(p)T}_{f1}, \tilde{\Theta}^{(p)T}_{f2}, \ldots, \tilde{\Theta}^{(p)T}_{fq}]^T$ with $\tilde{\Theta}^{(p)}_{fk} \in \mathbb{R}^{l_mN_f}$, $k \in \mathcal{Q}$. Let $H = \sum_{k=1}^{q}H_k$, and $\mathfrak{G}(s)$ is the transfer function of a



$$\begin{bmatrix} -5 & 5 & 5 \\ 5 & -5 & 5 \\ 5 & 5 & -5 \\ -5 & -5 & -5 \\ 5 & 5 & 5 \\ -5 & -5 & 5 \\ -5 & 5 & -5 \\ 5 & -5 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

Fig. 2. Cubic nominal formation of 9 agents and corresponding configuration matrix. Label leaders as pentagrams $\{1, 2, 3, 4\}$ and followers as squares $\{5, 6, 7, 8, 9\}$. The dashed edges are those ones that cannot be seen from the front view.

multi-input multi-output system as

$$\mathfrak{G}(s) := [I, \ldots, I]^T (sI - \Gamma - H)^{-1} [H_1, \ldots, H_q]. \tag{15}$$

Then, we define the time delay operator as $\delta_{\tau k}(y) := y(t) - y(t - \tau_k(t))$, and $\Delta = \mathrm{diag}(\delta_{\tau 1}, \delta_{\tau 2}, \ldots, \delta_{\tau q})$, $\delta_{\tau k}$ is the casual operator with bounded gain. Let w as the frequency of the signal after the Fourier transforms in the time domain, and the theorem of convergence is provided as follows.

Theorem 4. *Under Assumptions 1–3, if the leaders move along the polynomial trajectories $y^*_\ell(t)$, then the tracking error $\hat{\Theta}_f(t)$ of the followers under the control law (12) converges globally and exponentially to zero if the following two conditions are satisfied:*

(i) all the eigenvalues of $\Gamma + H$ have negative real parts;
(ii) the upper bound for the time-varying delays satisfies $\tau_0 < \left(w \cdot \max_{h\in\mathcal{H}} \sum_{g=1}^{\alpha_h} |(\iota w - \gamma_h)^{-g}| \cdot \sum_{l=0}^{l_m-1} k_l \cdot \rho_{\max}\right)^{-1}$ for $\forall w \in (0, \infty)$, where $\Gamma + H$ has $l_m N_f$ eigenvalues, we denote $\mathcal{H} = \{1, 2, \ldots, l_m N_f\}$, and for $h \in \mathcal{H}$ such that γ_h is the eigenvalue of $\Gamma + H$, ι is the imaginary unit, α_h is the algebraic multiplicity of γ_h, $\rho_{\max} = \|A_f\|_\infty = \max_{(N_\ell+1)\le i\le N} \sum_{j=N_\ell+1}^{N} |a_{ij}|$.

Proof. First, we analyze the stability of $\Gamma + H$ since the system must be stable if $\tau_k(t) = 0$. Under Assumptions 1–2, all the eigenvalues of $\bar{L}^s_{ff}$ have positive real parts. It can be seen that $\Gamma + H$ has no zero eigenvalue if $\bar{L}^s_{ff}$ has no zero eigenvalue. Besides, if condition (i) holds, all the eigenvalues of $\Gamma + H$ have negative real parts. As the same analysis in Theorem 2, the system with zero delay is exponentially stable.

Then, we investigate whether the tracking errors at non-zero frequency (i.e., $w \in (0, \infty)$) would decay to zero. Let Λ be the Jordan form of $\Gamma + H$, then

$$\begin{aligned}
\|\mathfrak{G}(s)\Delta\| &\le \left\| [I, \ldots, I]^T (sI - \Lambda)^{-1} [sH_1, \ldots, sH_q]\Delta \circ \frac{1}{s} \right\| \\
&\le \left\| [I, \ldots, I]^T (sI - \Lambda)^{-1} [sH_1 \delta_{\tau 1} \circ \frac{1}{s}, \ldots, sH_q \delta_{\tau q} \circ \frac{1}{s}] \right\| \\
&\le \sup_w \left\{ \max \left\|(sI - \Lambda)^{-1} sH\right\| \max_{k\in\mathcal{Q}} \left\| \delta_{\tau k} \circ \frac{1}{s} \right\| \right\}.
\end{aligned}$$

According to the definition of off-diagonal matrix A_f, we can obtain that $\|H\|_\infty = \sum_{l=0}^{l_m-1} k_l \cdot \rho_{\max}$. Besides, $\|(sI - \Gamma - H)^{-1}\|_\infty = \max_{h\in\mathcal{H}} \sum_{g=1}^{\alpha_h} |(s - \gamma_h)^{-g}|$, where $\gamma^{l_m} - k_{l_m-1}\lambda_i \gamma^{l_m-1} - \cdots - k_1 \lambda_i \gamma - k_0 \lambda_i = 0$ for $i \in \mathcal{V}_f$, and λ_i is the eigenvalue of L^s_{ff}.

With the small-μ stability theorem and Lemma 1 in Kao and Rantzer (2007), if the time delay system is stable for $\tau_k \in [0, \tau_0]$, the following inequality should be guaranteed for $w \in (0, \infty)$

$$\max_{h\in\mathcal{H}} \sum_{g=1}^{\alpha_h} |(\iota w - \gamma_h)^{-g}| \cdot \sum_{l=0}^{l_m-1} k_l \cdot \rho_{\max} < \frac{1}{w\tau_0}, \tag{16}$$

thus condition (ii) holds, and it is not necessarily satisfied at $w = 0$. By the terminal value theorem of Laplace transforms, it is readily seen that $(\Gamma + H)\tilde{\Theta}^{(p)}_f = 0$ when $s = \iota w = 0$ as $w = 0$. Since the nonsingularity of $\Gamma + H$ is satisfied under condition (i), it implies that the tracking error of the followers would converge to zero at $w = 0$. Here the proof completes. □

Remark 5. Similar to the method used in Theorem 4, a delay-dependent condition for the time-invariant delay system can also be proved, and the proof is omitted due to the page limit.

4. Implementation and simulation

We next provide two simulation examples to verify our proposed theoretical approaches.

4.1. Affine formation maneuvers without time delays

In the first example, a cubic nominal formation $(\mathcal{G}, r)$ of 9 agents in $\mathbb{R}^3$ are as illustrated in Fig. 2. The leader number meets $4 = d+1$ and their positions are not coplanar, and the underlying $\mathcal{G}$ is 4-rooted. Thus, Assumptions 1 and 2 are satisfied. We can obtain a signed Laplacian L^s with $\lambda(L^s) = \{0, 0, 0, 0, 1, 1, -0.5, 4\}$, and $\mathrm{rank}(L^s) = 4$. The stabilizing matrix D is calculated as $\mathrm{diag}(1, 1, 1, 1, 1, 1, 1, -1, 1)$ by Algorithm 1 in Xu et al. (2018).

The simulated results of the first example are shown in Figs. 3–4. The quintic polynomials are chosen for the leaders since a quintic polynomial in each axis has six coefficients that make it satisfy the six boundary conditions on initial and terminal position, velocity and acceleration for each leader. As can be seen in Fig. 3, the cubic formation maintains maneuvering to alter its centroid, scale, rotation, and other geometric patterns via affine transformation to dodge obstacles, e.g., circling the sphere and passing through a narrow corridor. The 2-norm form of tracking error can quickly converge to zero as shown in Fig. 4, and maintain nearly zero throughout the whole maneuvering process.

4.2. Affine formation maneuvers with time delays

In the sequel, we take a triangle nominal formation of 10 agents for the second simulation example, which is as shown in Fig. 5. Note that the configuration of nominal formation is not generic because there exist collinear agents, but three leaders are not collinear. Non-uniform time-varying delays satisfying Assumption 3 are set for each edge, the maximum time delay is $\tau_0 = 0.5$ s and maximum derivative of time delay is 0.9. The corresponding control gain calculated by Theorem 3 as $\{3.0000, 13.1439, 27.2935, 34.2250, 27.2935, 13.1439\}$. The simulated formation of agents in Fig. 6 can maneuver from random positions to a collinear shape. The tracking error converges to zero relatively slow and contains a period of oscillation in the initial part, which is caused by the non-uniform delays. Given the same control gain, if selecting the larger τ_{ij}, no maneuvers can be acted.

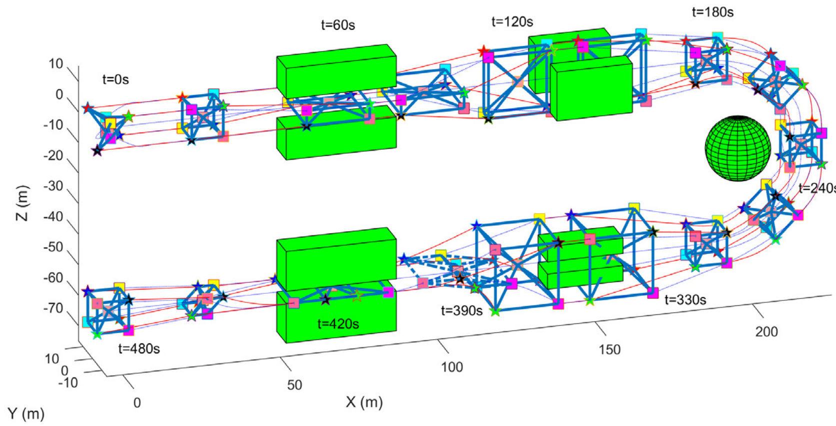


Fig. 3. A simulation example in Fig. 2 to illustrate time-varying affine formation maneuvers of double-integrator followers tracking quintic polynomial trajectories under control law (3), where $K = [1.2500, 5.4766, 11.3723, 14.2604, 11.3723, 5.4766]$. The arrows of lines are omitted for the convenience of viewing.

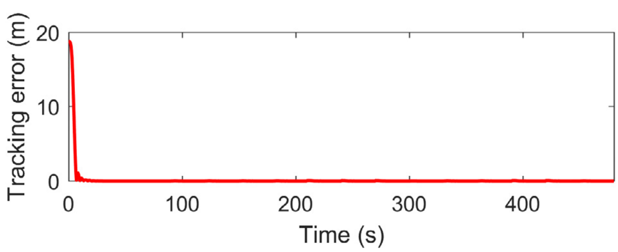


Fig. 4. Simulated tracking error of followers in Fig. 3.

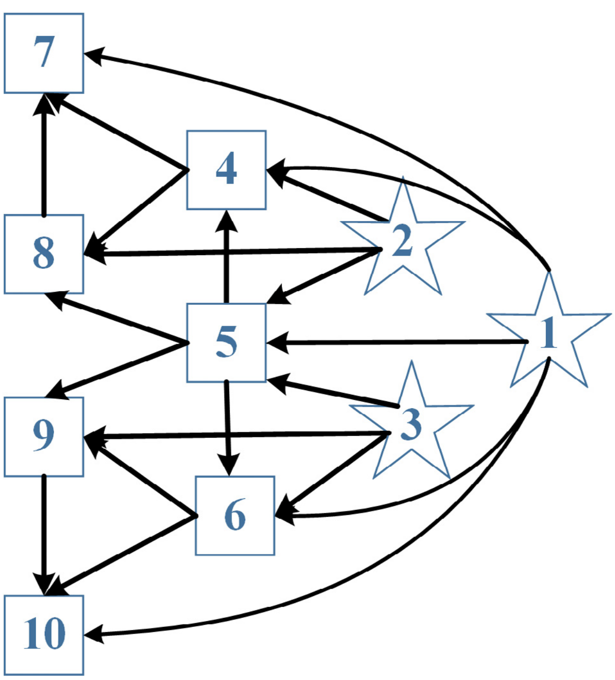


Fig. 5. Triangle nominal formation of 10 agents. Leaders are labelled as pentagrams {1, 2, 3}, and followers are labelled as squares {4, 5, 6, 7, 8, 9, 10}. The curved edges represent the ones that are collinear with other edges.

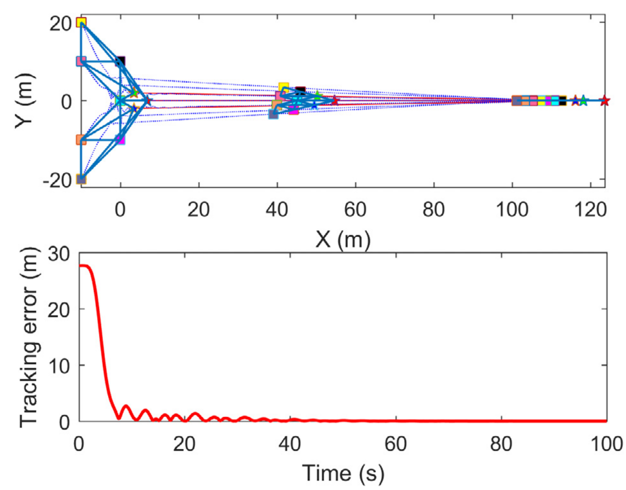


Fig. 6. A simulation example in Fig. 5 to illustrate time-varying affine formation maneuvers of fifth-order followers tracking cubic polynomial trajectories under time delay control law (12) and simulated tracking error of followers.

5. Conclusions

This paper proposes an innovative affine formation maneuver approach based on the directed graphs. Our method can control the followers with arbitrary-order integral dynamics to track arbitrary-order polynomial trajectories of the leaders in arbitrary dimensions successfully. As a consequence, time-varying formation maneuvers with the centroid, rotation, scales in different directions, as well as other geometric patterns satisfying the affine transformation, can be realized continuously. Compared with the requirement of acceleration feedback for the time-varying states of leaders in Zhao (2018), our proposed control laws in this paper do not need the higher-order information and save costs of installing additional sensors onboard. Control schemes for existing non-uniform time-varying delays are also designed for the sake of piratical implementations, since the delays are always regarded as the inherent characteristic of networked systems. Here we list several important topics for future research. For instance, the results presented in this paper may consider complicated underactuated dynamics and motion constraints.

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